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Abstract — The time-domain radiating characteristics of planar aperture antennas are obtained. The explicit formulas are given to describe the radiation field of circular and rectangular plane apertures at any point of the half space in front of aperture. We discuss the time-domain behavior of the radiation field for different space regions being of interest.

В работе получены точные выражения (в приближении интеграла Кирхгофа) для импульсных переходных характеристик круглой и прямоугольной плоских апертур для любой точки полупространства перед апертурой. Обсуждаются временные зависимости излучаемого поля на различных расстояниях от апертуры.

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Introduction

Properties of antennas operating with wide-band signals must be characterized by either the radiation field spatial-frequency dependence, or by spatial-temporal dependence, the latter being a Fourier transformation of the former one. The dependence can be considered as a set of pulse radiating characteristics (PRC) for each point of space around the antenna [1]. Using PRC, one can easily analyze, first of all, the field of antenna excited by ultra-short (without the carrier) pulses. In practice, a need in such an analysis occurs, for example, in measuring the characteristics of wide-band antennas by the method of near-zone field[2]. Besides, as it will be shown below, the calculation of PRC of the aperture antenna is considerably simplier than that for the antenna field at the fixed frequency, and the form of PRC can be easily interpreted.

Basic formulas

For the aperture antenna (mirror or the antenna array), we present PRC $E_a(t,\vec{r})$ at the point with the coordinate defined by the radius-vector $\vec{r}$ in the form of a convolution:

$$E(t,\vec{r}) = h_a(t) * E_a(t,\vec{r}), \tag{1}$$

where $h_a(t)$ is the PRC of the feed or PRC of each element of the antenna array (the latters are accepted identical for simplicity), $E_a(t,\vec{r})$ can be defined as PRC of the antenna aperture:

$$E_a(t,\vec{r}) = \iint\limits_{S_a} g(\vec{r}_a)\,\delta\left(t - |\vec{r}-\vec{r}_a|/c\right) \frac{d^2\vec{r}_a}{|\vec{r}-\vec{r}_a|}, \tag{2}$$

where $\vec{r}_a$ is the radius-vector of the point on aperture, S_a is the region occupied by the aperture, $g(\vec{r}_a)$ is the distribution of amplitudes of radiating elements over the aperture, $\delta(t)$ is the Dirac delta function. Since in (1) just the term $E_a(t,\vec{r})$ defines the fundamental peculiarities of spatial-temporal structure of the antenna field, further we restrict ourselves by the analysis of (2)

The limits of application of (2) can be estimated by the Fourier transformation $E_{a,\omega}(\vec{r})$.

$$E_{a,\omega}(\vec{r}) = \int E_a(t,\vec{r})e^{+i\omega t}dt = \iint\limits_{S_a} \frac{g(\vec{r}_a)e^{ik|\vec{r}-\vec{r}_a|}}{|\vec{r}-\vec{r}_a|} d^2\vec{r}_a, \qquad (3)$$

$k = \omega/c$. Expression (3) is the antenna field at the frequency ω, calculated by the aperture method [4] with an accuracy up to the constant factor at the fixed frequency of the multiplier. The aperture method is widely used at least for qualitative description of the antenna field (without taking into account variations of the electric field vector orientation at large angles between the normal to aperture and the direction to the observation point D_a) in the case, when the dimension of the aperture D_a much larger than the wavelength, i.e. for the frequencies $\omega \gg 2\pi c/D_a$. In order this requirement be fulfilled, we shall assume, that values of the spectral components of PRC $h_a(t)$ out of the region of frequencies $|\omega| \gg 2\pi c/D_a$ are negligibly small, so that in expression (1) for PRC of the antenna "incorrect" spectral components (2) will be cut off.

We note, that qualitative dependence $E_{a,\omega}(\vec{r})$ on $\vec{r}$ for inphase antennas at the fixed frequency ω in the near zone was investigated in details (see, for example, [3,4]): the field is of essentially different character in so called projector region - geometrical continuation of the aperture in the direction of the normal to the phase front and outside the projector beam - in the region of side lobes. However, expressions for $E_{a,\omega}(\vec{r})$ are rather cumbersome. Even for apertures of the simplest form (circular, rectangular) at distances admitting the expansion $|\vec{r}-\vec{r}_a|$ into power series with conservation of only quadratic terms - projections $\vec{r}_a$ (Fresnel approxmation), the field is presented either by expansion into special functions for the circular plane aperture, or by a set of Fresnel integrals for the rectangular aperture [5].

In constract to (3) calculation of the aperture PRC is simplier and, as it will be shown below, the result in many cases can be represented in elementary functions. To calculate the PRC let us take a simply obtained integration formula for the integral, containing δ - function of the complex argument:

$$\int\int\limits_{S} f(x,y)\delta[\varphi(x,y)]dxdy = \int\limits_{\Gamma} \frac{f(x(\gamma),y(\gamma)}{|\,grad\varphi\,|, x = x(\gamma), y = y(\gamma)} d\gamma, \qquad (4)$$

where Γ is the curver determined from equation $\varphi(x,y) = 0$; $x = x(\gamma), y = y(\gamma)$ is a parametric representation of Γ, dγ is an element of length Γ. It is supposed here, that the solution of equation $\varphi(x,y) = 0$ for $x, y \subset S$ exists and determines unique curve Γ. If for all $x, y \subset S$ $\varphi > 0$ or $\varphi < 0$, then integral (4) equals to thero. For integral (2) the equation determining curve Γ, is

$$|\vec{r} - \vec{r}_a| = ct. \tag{5}$$

In three-dimentional space (5) describes a sphere with a center at point $\vec{r}$ and radius ct. The sphere crosses the aperture plane at $ct > z$, where z is adistance from point $\vec{r}$ to aperture plane (for $ct < z$, it is obvious that $E_a = 0$). Curve Γ (a locus of intersection of the sphere and the aperture plane) is a circle with radius $b = \sqrt{(ct)^2 - z^2}$, with the center at point $\vec{\rho}$, where $\vec{\rho}$ is a projection of vector $\vec{r}$ on the aperture plane (see Fig.1). For this circle $|\vec{r} - \vec{r}_a| \, |grad(\frac{1}{c} |\vec{r} - \vec{r}_a|)|_{\vec{r}_a \subset \Gamma} = b/c$ from where

$$E_a(t,\vec{r}) = \begin{cases} 0, & (a); \\ \dfrac{c}{b} \displaystyle\int_{\Gamma_a} g(\vec{r}_a \subset \Gamma_a) d\gamma & (b); \end{cases} \tag{6}$$

Γ_a is a part of Γ, belonging to S_a. For the constant amplitude distribution over the aperture $g = 1$ in the case (b) for E_a it follows elementary formula:

$$E_a(t,\vec{r}) = c\phi, \tag{7}$$

where angle ϕ is given in Fig.1.

Such transformation in the physical sense can be explained by the next way. If each element of the aperture radiates δ–pulse at the moment $t = 0$, then, at the moment $t' > 0$ the field at the point $\vec{r}$ is defined only by elements lying on the circle or its part, where the sphere of ct' radius and center at $\vec{r}$ point crosses the aperture plane. The field amplitude is defined by the suspended integral over the given circle; the latter in many cases can be expressed by elementary functions.

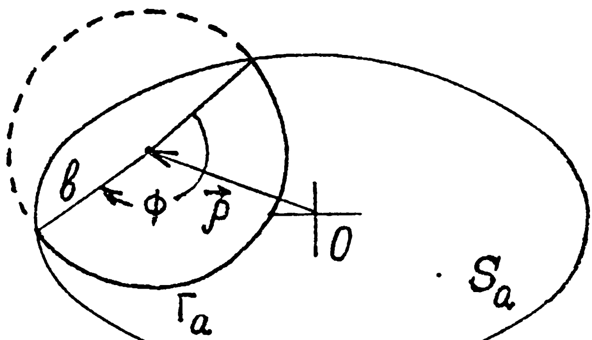

Fig.1. The curve of intersection of the sphere with radius *ct* and the aperture plane. Here S_a - antenna aperture.

PRC of circular plane aperture

Introducing the cylindrical reference system ρ, φ, z, the center of which coincides with the center of the circular aperture of a radius, and for simplicity we assume $g(\vec{r}_a) = 1$. Using (6), and taking into account, that the field is dependent only on t, z, ρ $\quad (\rho = \mid \vec{\rho} \mid)$, integration, we obtain $E_a(t,\rho,z) = 2c\Big(\pi - \phi(t)\Big)$, we obtain in the limits of the projector area, i.e. at $\rho < a$:

$$E_a(t,\rho,z) = \begin{cases} 0, & 0 < ct < z; \\ 2\pi c, & z < ct < \sqrt{z^2 + (a-\rho)^2}; \\ 2c\left(\pi - \arccos\dfrac{a^2 - \rho^2 - b^2}{2\rho b}\right), & \sqrt{z^2 + (a-\rho)^2} < \\ & < ct < \sqrt{z^2 + (a+\rho)^2}; \\ 0, & \sqrt{z^2 + (a+\rho)^2} < ct. \end{cases} \tag{8}$$

Outside the limits of the projector area, i.e. at $\rho > a$:

$$E_a(t,\rho,z) = \begin{cases} 0, & 0 < ct < \sqrt{z^2 + (\rho - a)^2}; \\ 2c\arccos\dfrac{-a^2 + \rho^2 + b^2}{2\rho b}, & \sqrt{z^2 + (\rho - a)^2} < \\ & < ct < \sqrt{z^2 + (\rho + a)^2}; \\ 0, & \sqrt{z^2 + (a+\rho)^2} < ct. \end{cases} \tag{9}$$

Diagrams of these functions for different values of ρ, z are given in Figure 2,3. When $\rho = 0$, i.e. at the axis z $E_a(t,\rho,z)$ has the form of a rectangle, here with increase of z the delays of leading and trailing edges are changed with different velocity. It can be seen, that with small z the leading edge delays $\sim z$, and the delay of the trailing edge is practically unchanged, with further increase of z, the velocity of the PRC duration variation is decreased. With increase of the distance from the aperture axis (with increase of ρ) in the limits of the projector area (see Fig. 3) the leading edge of the pulse remains

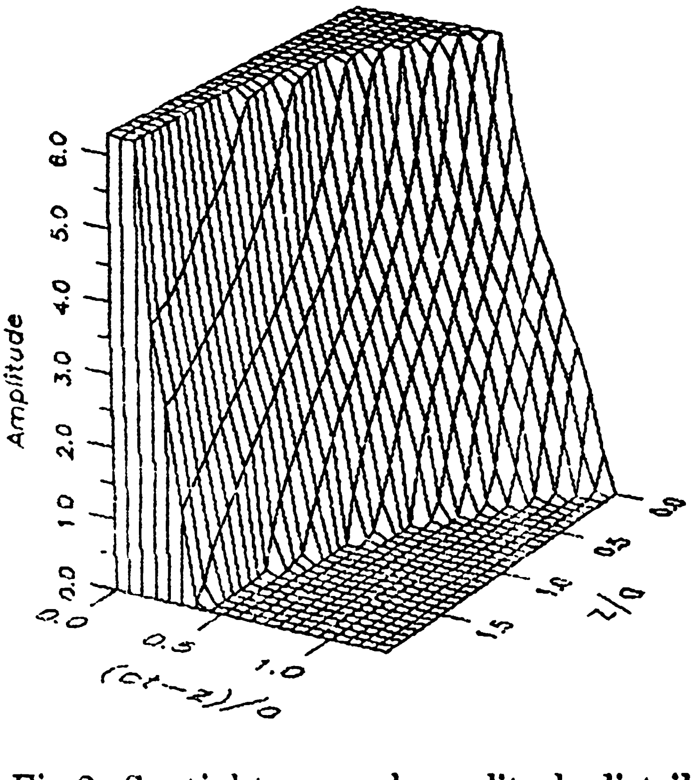

Fig.2. Spatial-temporal amplitude distribution $E(t, \rho = a/4, z)$ for circular plane aperture.

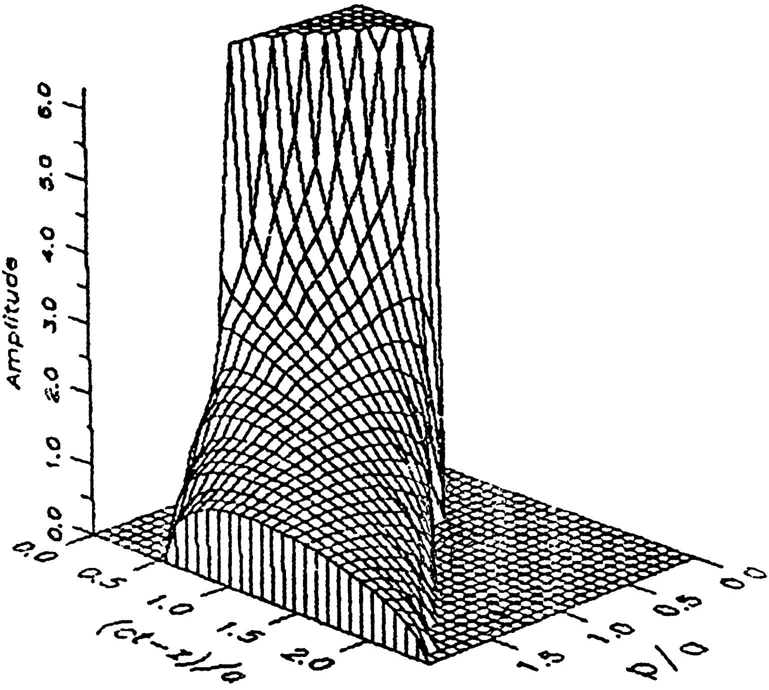

Fig.3. Spatial-temporal amplitude distribution $E(t, \rho, z = a/2)$ for circular plane aperture.

unchanged, the duration of the pulse leading edge is increased, here the decrease of the plane part length at the top of the pulse and its broadening take place simultaneously. With increase of ρ outside the limits of the projector area the pulse edges are broadened, the amplitude drops and the duration tends to the value $2a$. With increase of z at $\rho < a$ the pulse duration is decreased.

Outside the projector area the steepness of the leading edge is larger than the trailing one, these are connected with nonlinear dependence of the *arccos* function argument on time.

At the infinity the field at each moment of time will be defined as the integral over the line of two plane intersection, the first is the aperture plane and the second (the sphere of radius $r, r \to \infty$) is the plane inclined by the angle θ to the aperture plane and is given in the form: $E_a(t, \theta, \rho) \to \frac{1}{r} f(t', \theta), r \to \infty$, where $z = r\cos\theta, \rho = r\sin\theta, ct' = ct - r$,

$$f(t', \theta) = \begin{cases} 0, & a\sin\theta < |ct'|; \\ \dfrac{2c}{\sin^2\theta}\sqrt{(a\sin\theta)^2 - (ct')^2}, & |ct'| < a\sin\theta. \end{cases} \tag{10}$$

For $\theta \to 0$ $f_\delta(t', \theta) \to \pi a^2 \delta(t')$.

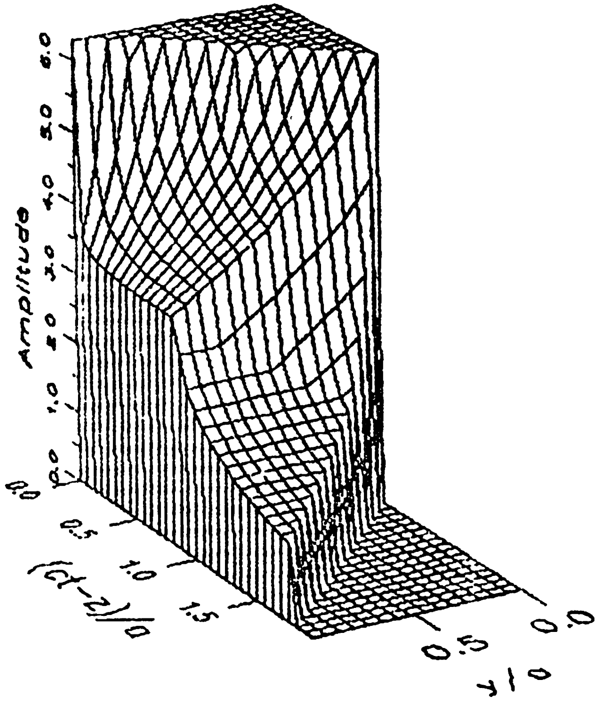

Fig.4. Spatial-temporal amplitude distribution $E(t, x = 0, y < a, z = a/8)$ for square aperture.

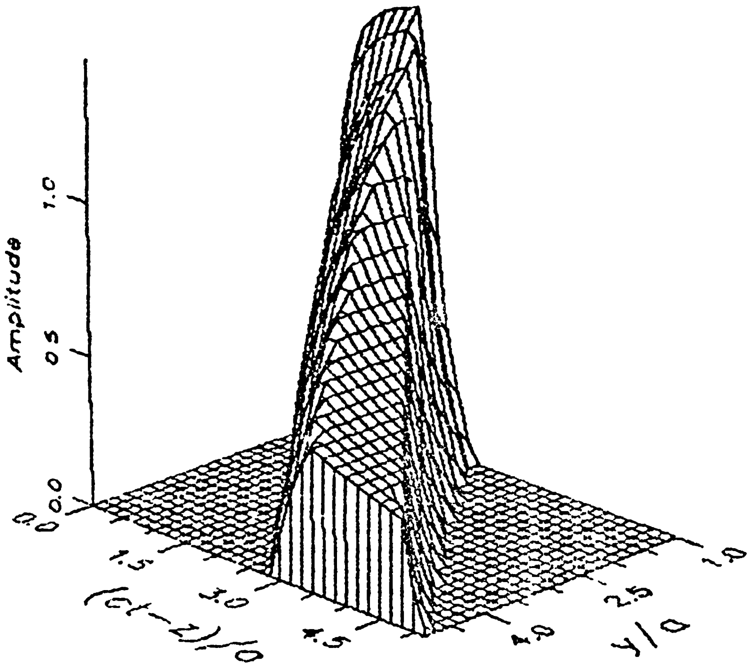

Fig.5. Spatial-temporal amplitude distribution $E(t, x = 0, y > a, z = 0.625a)$ for square aperture.

PRC of rectangular plane aperture

We obtain an expression for the time dependence of the rectangular plane aperture field in the limits of the projector area if we change the limits of integration over φ in expression (??)

$$E_a(t,x,y,z) = 2\pi - \sum_1^8 \phi_{m,n}\,, \tag{11}$$

$$\phi_{mn} = \begin{cases} 0, & ct < \sqrt{z^2 + l_m^2}; \\ \arccos \dfrac{|l_m|}{\sqrt{(ct)^2 - z^2}}, & \sqrt{z^2 + l_m^2} < ct < \sqrt{z^2 + l_m^2 + l_n^2}; \\ \arccos \dfrac{|l_m|}{\sqrt{l_m^2 + l_n^2}}, & \sqrt{z^2 + l_m^2 + l_n^2} < ct; \end{cases} \tag{12}$$

$m = 1 \ldots 4$ is the number of the aperture boundary; n is the number of the boundary perpendicular to that with the number m, l_m, l_n is the distance from the projection of the observation point on the aperture plane up to the boundaries of apertures with numbers m, n, respectively. The form of the field $E_a(t, x = 0, y < a_y, z)$ for $a_x = a_y$ is given in Figure 4. In contrast to PRC of circular plane aperture at $\rho = 0$, the duration of the trailing edge is always not zero and contains several zones defined by different character of intersection between the circles and the aperture boundary. With increase of the distance from the axis, the duration of the plane zone at the top of PRC is decreased, and the total duration of the pulse is increased. The form of the trailing edge for the rectangular aperture will depend also on the relation of a_x/a_y. We give expressions for the field outside the limits of the projector area for two cases: when only one projection is outside the aperture limits (13) and when both projections are outside the aperture limits (see Figure 5, expression (17)).

$$E_a(t,x,y,z) = \phi_{right} + \phi_{left}, \tag{13}$$

$$\phi_{right,left} = |\phi_2 - \phi_1|, \tag{14}$$

$$\phi_1 = \begin{cases} 0, & ct < \sqrt{z^2 + l_i^2}; \\ \arcsin \dfrac{|l_i|}{\sqrt{(ct)^2 - z^2}}, & \sqrt{z^2 + l_i^2} < ct < \sqrt{z^2 + l_i^2 + l_{j,n}^2}; \\ 0, & ct > \sqrt{z^2 + l_i^2 + l_{j,n}^2}. \end{cases} \qquad (15)$$

$$\phi_2 = \begin{cases} 0, & ct < \sqrt{z^2 + l_k^2}; \\ \arcsin \dfrac{l_i}{\sqrt{(ct)^2 - z^2}}. & \sqrt{z^2 + l_k^2} < ct < \sqrt{z^2 + l_k^2 + l_j^2}; \\ \arccos \dfrac{l_j}{\sqrt{(ct)^2 - z^2}}, & \sqrt{z^2 + l_k^2 + l_j^2} < ct < \sqrt{z^2 + l_i^2 + l_j^2}; \\ 0, & \sqrt{z^2 + l_i^2 + l_j^2} < ct; \end{cases} \qquad (16)$$

where $\phi_{right,left}$ are angles in the right-hand and left-hand of half-plane, the boundaries of half-planes are defined by the normal to two boundaries passing through the projection of the observation point, l_k, l_i is the distance from the projection of the observation point on the aperture plane up to the nearest aperture boundary and to the parallel one; j, n is the number of the aperture boundary perpendicular to that with number i.

$$E_a(t, x, y, z) = \phi_{max} - \phi_{min}, \qquad (17)$$

where $\phi_{max} > \phi_{min} > 0$, two others $\phi_m = 0$

$$\phi_m = \begin{cases} 0, & ct < \sqrt{z^2 + l_m^2 + l_n^2}; \\ \arccos \dfrac{l_m}{\sqrt{(ct)^2 - z^2}}, & \sqrt{z^2 + l_m^2 + l_n^2} < ct < \sqrt{z^2 + l_m^2 + l_k^2}, \\ & \text{for } m = 1, 2; \\ \arcsin \dfrac{l_m}{\sqrt{(ct)^2 - z^2}}, & \sqrt{z^2 + l_m^2 + l_n^2} < ct < \sqrt{z^2 + l_m^2 + l_k^2}, \\ & \text{for } m = 3, 4; \\ 0, & \sqrt{z^2 + l_m^2 + l_k^2} < ct; \end{cases} \qquad (18)$$

where l_m is the distance from the projection of the observation point up to one of the aperture boundaries with number m; $m = 1 \ldots 4$ is the number of the aperture boundary; n, k are the numbers of boundaries perpendicular to that with number m, here $l_k > l_n > l_m$.

At infinity the field at each moment of time will be defined as the integral over the line of two plane intersection, the first is the aperture plane and the plane (the sphere of the infinite radius) inclined by the angle θ to the aperture plane (polar angle) and will be presented in the form: $E_a(t, \theta, r, \varphi) \to \frac{1}{r} f(t, \theta, \varphi)$, where φ- azimuth angel, θ- polar angle (here $x = r \sin\theta \cos\varphi$, $y = r \sin\theta \sin\varphi$, $z = r\cos\theta, ct' = ct - r$, it being known that $a_x \sin\theta\cos\varphi < a_y \sin\theta \sin\varphi$, $\sin\theta\cos\varphi > 0, \sin\theta\sin\varphi > 0$).

$$f(t,\theta,\varphi) = \begin{cases} \dfrac{2a_y c}{\sin\theta\cos\varphi}, & \sin\theta(a_x\cos\varphi + a_y\sin\varphi) < |ct'|; \\ \dfrac{c(a_x\cos\varphi - a_y\sin\varphi - |ct'|)}{\sin\theta\sin\varphi\cos\varphi}, & \sin\theta(a_x\cos\varphi - a_y\sin\varphi) < |ct'| < \\ & < \sin\theta(a_x\cos\varphi + a_y\sin\varphi); \\ 0, & \sin\theta(a_x\cos\varphi + a_y\sin\varphi) < |ct'|. \end{cases} \tag{19}$$

Conclusion

PRC for circular and rectangular plane apertures have been obtained for all points of half space in front of the aperture. The formulas obtained are turned out to be simplier than those known for aperture antenna fields in the case of monochromatic signal they can be used for calculation of aperture antenna spatial-frequency characteristic.

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