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**STATISTICAL HUBBLE DIAGRAM OF THE UNIVERSE**

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### Abstract

The statistical Hubble diagram constructed according to an average luminosity of sources is used for various cosmological theories verification. For every little red-shift bit from  $z$  to  $z+\Delta z$ , the average luminosity is determined by means of statistically representative number of galaxies corresponding to a normal (gaussian) distribution function. The total number of sources used consist of approximately 9000 galaxies and 4000 quasars. This gives Hubble diagram as regression function  $m(z) = 2.83 \log(z)+18.6$ , and allows one to determine the relation between apparent luminosity  $L(z)$  and distance  $R(z)$  as  $R(z) = 10^{3.72} z^{0.56} L(z)^{1/2}$ . If the average luminosity  $L(z)$  does not vary we have the relation  $R(z) \sim z^{0.56}$  discordant with the relation of standard cosmological theory. Otherwise, if  $L(z) = L_0 z^{0.88}$  we get the usually used relation  $R(z) \sim z$ , corresponding to the standard cosmology. But we prove that this dependence requires too high luminosity evolution which does not satisfy the reality. Thus, it is necessary to exclude the considerable evolution and use approximately  $R(z) \approx 830 z^{0.56}$  Mpc. This result leads to the revision of distance estimations for galaxies and quasars in the Universe, for example, for  $z = 1$  true distances are 3-4 times less than those given in the standard cosmology. Our approach allows to solve some serious problems of theory. For example, superluminal velocity of quasar motion becomes an underluminal one, and the average quasar luminosity becomes not more than the average luminosity of galaxies.

## Introduction

For common properties of the Universe investigation through its luminous component the main role plays the Hubble diagram, i.e. the dependence of the apparent luminosity  $m(z)$  of objects on their red-shift value  $z$ . According to the previous idea produced by Hubble in 1953 the comparison between this dependence and the theoretical one can define the true model of the Universe. Hubble considered this program as the main one, but he did not implement it in reality. In 1956 his followers Sandage and Hoyle specified it as a problem of determination of deceleration parameter  $q$  (Hoyle, 1992).

The method offered by Sandage used the selection of galaxies with standard absolute luminosity  $L(z)$  corresponding to different red-shifts  $z$ . The criterion used the first-ranked sources in a cluster of galaxies. As a result Sandage (1961) got a diagram up to values  $z < 0.3$ , which satisfies the theoretical straight line, the inclination  $\frac{dm}{d \log(z)}$  equals to 5. This is indeed the first corroboration of so called standard cosmological theory. Attempts to determine the deceleration value  $q$  failed by the absence of data for  $z > 1$ , where the theoretical dependencies  $m(z, q)$  differ significantly for various  $q$  values. However recent investigations for large  $z$  values show also the impossibility of deceleration parameter  $q$  determination because the error exceeds the determined value. In the 60-ths a hope arose to decide this problem by using quasar data with red-shifts up to  $z=(2-3)$ . But  $m(z)$  data for quasars both had a large dispersion and were located in  $m(z)$  plane on a considerable distant from the galaxy line. These data correspond to a straight line with less than 5 inclination in the  $m(z)$  graph. Wampler (1987) made an attempt to select quasars by using Baldwin relation which connects the absolute luminosity of quasar with the width of radiation lines of four times ionized carbon. This approach allows to increase the derivation  $\frac{dm}{d \log(z)}$  for quasar up to 5, but the line does not correspond to galaxy one obtained by Sandage. To avoid the discrepancy in that paper a segment of quasars curve was simply moved on  $6^m$  in order to form united  $m(z)$  dependence for galaxies and quasars, without any other arguments. As a result, this method gives for  $q$  value so wide range  $0.5 < q < 3$ , that this means the failure of deceleration parameter determination. The situation allowed Burbidge (1989) to conclude that the method used

is unsatisfactory for solving the considered problem. The equation for an apparent luminosity  $E$  in physical units is

$$E = \frac{L(z)}{R^2(z,q) (z+1)^2}, \quad (1)$$

where  $L(z)$  is the absolute luminosity of a source,  $R(z,q)$  is the theoretical metrical distance. For the theory verification it is necessary first of all to determine experimentally  $R(z)$  function. The measurement of  $R(z,q)$  function may be fulfilled using expression (1) if the standard sources (standard candles) are chosen i.e. the sources with equally luminosities and various  $z$  values. In the considered case the expression depends on  $R(z)$  function only, and, consequently, it can be measured experimentally. Some serious investigators believe till now that Sandage's results confirm the validity of the standard theory of the expanding Universe.

But Sandage's selection method evokes some serious doubts. Indeed, it uses the hypothesis that there exists an upper limit of the luminosity, and by selecting first-ranked galaxies we get sources with an equal maximal luminosity only. But the luminosity is a statistical value with the normal (gaussian) probability distribution function. By selecting the first-ranked sources we pick out the galaxies from the tail of this function. In this case it is impossible to provide the choice of standard sources, because for different  $z$  values we use galaxies from various parts of the distribution function with principally different character of evolution. By using Sandage's method only exotic galaxies are picking out which luminosity is much more than the average one, and it is naturally that they evolve more rapidly than galaxies at the distribution function maximum. This statement is confirmed by a test selection of bright and weak galaxies from UGC catalogue at determined  $z$  values. It gives the Hubble curve inclination  $\frac{dm}{d \log(z)} = 4.0$  for the first case and  $\frac{dm}{d \log(z)} = 1.7$  for the second one. Thus, an inadequate galaxy selection spoils the Hubble curve. The true result can be found by using the galaxies which luminosity locates at the maximum of a normal distribution function. This value determines an average value of luminosities of galaxies located in the interval of  $z$  values from  $z$  to  $z+\Delta z$ . The statistical approach used considers every luminous source as an accidental realization of the standard candle in the Universe. It is correct, if every  $[z, z+\Delta z]$  interval contains a statistically representative ensemble of luminous sources. The

method described is named in mathematics as a regression analysis. It is used in the scientific investigation for the search of regularities hidden in noises, and when the experimental data dispersion is large. It is the exclusive way for objective statistical regularities from sequences of casual data like  $m(z)$  ones. That is why the regression function  $m(z)$  is used here for the comparison with cosmological theories.

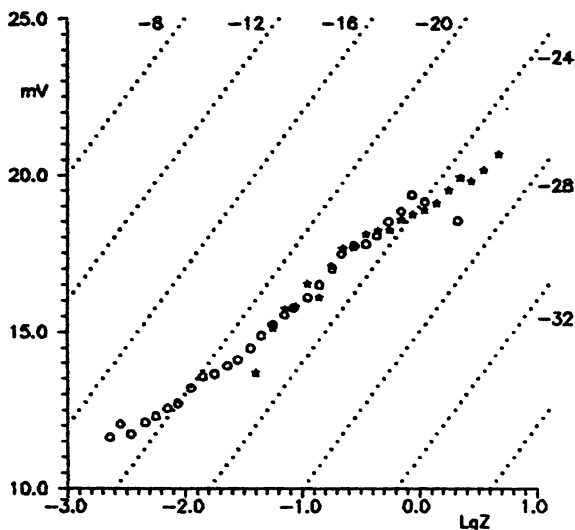
At first our attempt of the Hubble diagram determination concerned quasars. We used the catalogue (Hewitt and Burbidge, 1987) which contains  $m$  and  $z$  values for approximately 3500 quasars. As a result, the average  $m(z)$  curve of quasars joined good in common part with the curve of galaxies formed early for 160 galaxies (Troitsky and Gorbacheva, 1989). This curve did not take into account  $K(z)$ -correction for quasars, and common curve was described by the function

$$m(z) = 5 \log(z) - 8 \log(z+1) + 21.5 \quad (2)$$

Later, we used two catalogues of quasars (Hewitt and Burbidge, 1987; Veron-Cetti and Veron, 1989) and the data for 1000 galaxies and  $m(z)$  dependence was successfully approximated by the same expression (2), (see Troitsky et al., 1992). The  $K(z)$ -correction also was not used here by the cause of its smallness, it equals  $(1.0-1.5) \cdot \log(z+1)$ . Thus, in present paper we use the statistical principle of Hubble diagram formation, which allows oneto reflect general properties of space-time of the Universe. The approach permits one to use the Hubble diagram for various theoretical models of the Universe testing, this is the very important goal of the observant cosmology.

## 1. Observational data and results

For Hubble diagram construction we use a number of surveys published making the comparison of every catalogue on compatibility with others. In general, we use the data of 9530 galaxies and of approximately 4000 quasars. We find data for quasars in papers (Hewitt and Burbidge, 1987; Veron-Cetti and Veron, 1989; Hewett et al., 1991). For data concerning galaxies we use papers: Tifft and Gregory (1988) - 82 galaxies, Huchra et al. (1983) - 2074 galaxies, Huchra et al. (1990) - 415 galaxies, Peterson et al. (1986) - 342 galaxies, Salzer and McAlpin (1988) - 103 galaxies, Chapman et al. (1988) - 257 galaxies, Ostriker et al. (1988) - 241 galaxies, Dressler and Shekman (1988) - 1269



F i g. 1

General regression Hubble diagram.  $\circ$  - galaxies,  
 $*$  - quasars, dotted line is the family of theoretical curves  
 $m(z, M) = 5 \log z + M + 43$  for  $q_0 = 1$ ,  $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$

Table I

$\log(z)$	number of sources	$\bar{m}_V$ with K(z) correction	K(z)	$\sigma(m_V)$
GALAXIES				
-2.64	120	11.63	0.003	1.147
-2.55	165	12.06	0.004	1.143
-2.46	318	11.74	0.005	1.386
-2.34	343	12.11	0.007	1.279
-2.25	328	12.31	0.0085	1.250
-2.15	328	12.56	0.01	1.229
-2.06	311	12.71	0.013	1.139
-1.95	393	13.20	0.02	1.154
-1.85	582	13.55	0.02	1.156
-1.75	916	13.65	0.027	1.168
-1.64	1092	13.91	0.03	1.135
-1.55	879	14.11	0.04	1.130
-1.44	819	14.48	0.05	1.139
-1.35	909	14.88	0.07	1.106
-1.25	383	15.23	0.08	1.151
-1.15	258	15.55	0.1	1.163
-1.07	115	15.76	0.12	1.144
-0.95	76	16.09	0.16	1.185
-0.85	59	16.49	0.2	1.177
-0.74	59	17.01	0.25	1.323
-0.66	47	17.48	0.3	1.007
-0.55	36	17.73	0.45	1.079
-0.45	26	17.79	0.75	1.029
-0.36	33	18.06	1.07	1.043
-0.26	7	18.51	1.48	1.072
-0.15	9	18.85	2.01	1.118
-0.06	10	19.37	2.5	1.030
0.05	14	19.16	3.18	1.396
0.17	2	19.67	4.04	1.697
0.39	5	18.09	5.88	0.688

Table II

$\log(z)$	number of sources	$\bar{m}_V$ with K(z) correction	K(z)	$\sigma(m_V)$
QUASARS				
-1.40	10	13.68	-0.025	0.923
-1.25	18	15.13	-0.036	1.240
-1.15	14	15.72	-0.045	1.163
-1.05	22	15.80	-0.056	1.014
-0.95	40	16.52	-0.069	1.502
-0.85	51	16.10	-0.086	1.163
-0.75	69	17.09	-0.107	1.330
-0.65	102	17.64	-0.13	1.404
-0.55	159	17.68	-0.16	1.310
-0.45	281	18.11	-0.20	1.161
-0.35	269	18.22	-0.24	1.039
-0.25	311	18.24	-0.29	1.068
-0.15	372	18.57	-0.35	1.047
-0.05	496	18.74	-0.42	1.042
0.05	599	18.89	-0.49	1.028
0.15	967	19.11	-0.57	1.135
0.25	1319	19.53	-0.67	1.033
0.35	2035	19.93	-0.77	1.113
0.45	587	19.81	-0.87	1.135
0.55	141	20.17	-0.99	1.349
0.65	9	20.67	-1.15	1.042



galaxies, Troitsky at al. (1992) - 1103 galaxies, Karachentsev and Kopylov (1990) - 313 galaxies, Karachentsev (1987) - 1054 galaxies, Kapahi at al. (1990) - 11 galaxies, Nilson (1973) - 1070 galaxies, Singal (1988) - 41 galaxies, Djorgovski and Spinrad (1981) - 24 galaxies, Sandage and Perelmuter (1991) - 161 galaxies, Smith and Hekman (1989) - 70 galaxies, Pedelty at al. (1989) - 12 galaxies. For calculation of the  $K(z)$ -effect of galaxies with red-shift  $z > 0.25$  we use average spectral index 4.0 according to paper (Troitsky at al., 1992). That corresponds to elliptical galaxies which are observed mainly with  $z > 0.25$ . As a result, we obtain the following expression for  $K(z)$  - correction:  $K(z) = 12.5 \log(z+1) - 0.9$

The general Hubble diagram  $m(z)$  with  $K(z)$ -corrections is shown in fig.1. It may be well approximated by function

$$m(z) = (2.83 \pm 0.1) \log(z) + 18.6 \quad \text{for } 10^{-3} < z < 4.5 \quad (3)$$

In this figure a family of theoretical  $m(z)$  curves corresponding to various absolute galaxy and quasar luminosity  $M$  for model  $q=1$  is shown. Numerical  $m(z)$  values and other data are given in table I for galaxies and in table II for quasars.

## 2. Analysis of possible systematic distortions of the Hubble diagram.

Main causes of distortion of regression dependence  $m(z)$  are the  $K(z)$ -effect and Malmquist bias. For defining the  $K(z)$  - correction for apparent luminosity, we make a special investigations of galaxy and quasar spectral form for various morphological types. With taking into account quantity of galaxy distribution we got their spectral parameters in the red-shift range  $10^{-3} < z < 0.5$ . As a result we got the following formulas and used them:

$$K_1(z) = 12.5 \log(z+1) - 0.9 \quad 0.24 < z < 0.52 \quad (4)$$

$$K_2(z) = 3.5 \log(z+1) \quad 0 < z < 0.25$$

By defining  $K(z)$ -correction for quasar we use an average spectrum for 300 quasars (Troitsky and Gorbacheva, 1992). The result is the quasar spectrum at the wavelength interval  $0.2 < \lambda < 0.55$  is well described by a linear function of  $\lambda$  corresponding to spectral index  $ds/d\lambda = -1.6$ . According to Hewitt and Burbidge catalog for quasars we get slightly less spectral index  $ds/d\lambda = -1$  (Troitsky and Gorbacheva, 1993). These data lead for the following expression for  $K(z)$ -effect  $0 < -K(z) < 1.5 \log(z+1)$ . By

building general  $m(z)$  curve for quasars from observed Hubble curve we subtract the correction

$$K(z) = - 1.5 \log(z+1) \quad (5)$$

which at  $z=4$  equals to only one magnitude, for further details see (Troitsky and Gorbacheva, 1992,1993). Malmquist selection effect is the following. With the red-shift increasing, a part of bright sources in data increases and a part of weak sources decreases due to a limited sensitivity of the telescope. This results in decreasing the inclination of the average curve  $dm(z)/d\log(z)$  in  $m(z)$  graph, but a quantitative calculation of this effect does not exist. We know only two attempts to make some estimations in papers (Neuman and Scott, 1961, 1978), which contain useful reasoning, but they do not contain strict calculations adequate to the physical problem. However, the effect is often referenced, especially in the cases when somebody wants to contest the data which do not keep within the framework of his theoretical conception.

For excluding this opportunity, we make a special calculations of the selection effect. They show that the change of the average luminosity disappears, if a telescope sensitivity exceeds on 2 the average  $m(z)$  value for every  $z$  considered (Troitsky and Gorbacheva, 1992). Maximal average  $z$  value in our  $m(z)$  does not exceed  $20^m$ , consequently, the telescope must have the sensitivity  $22^m$ , that is fulfilled. Another confirmation of the absence of Malmquist effect influence is the gaussian form of the luminosity distribution function for all  $z$  values with the same root mean square deflection  $\sigma(m)=1.0$  for both galaxies and quasars. Thus, we got the general Hubble diagram which is free from discussed above distortions which are conditioned by inadequate source selection.

### 3. The results and conclusions

Let us now compare the observed  $m(z)$  dependence with the theoretical one which depends on two unknown values: on photometric distance  $R(z)$  and on luminosity evolution function( $z$ ), i.e.

$$E_{th}(z) \sim \frac{L(z)}{R^2(z)} \quad (6)$$

The experiment gives the expression  $E_{exp}(z) = 10^{-0.4m(z)}$ , where

$m(z)$  is the general Hubble dependence according to (3). After substitution we have

$$E_{\text{exp}}(z) = 10^{-7.44} z^{-1.12} \quad (7)$$

From (6) and (7) we get the following experimental value of  $R(z)$

$$R_{\text{exp}}(z) = L(z)^{1/2} z^{0.56} 10^{3.72} \quad (8)$$

Thus, the test allows to determine ratio  $R(z)/L(z)^{1/2}$ . Let us consider two extreme cases:

a) luminosity does not vary  $L(z)=L_0 = \text{const}$ , then

$$R_{\text{exp}}(z) = L_0^{1/2} 10^{3.72} z^{0.56+\beta/2} = D_0 z^{0.56}, \quad (9)$$

here  $D_0$  is the distance to sources with  $z=1$ ;

b) the distance function corresponds to Hubble law  $R=R_0 z$ , where  $R_0 = \frac{c}{H_0} = 4 \cdot 10 \text{ Mpc}$  at  $H_0 = 75 \frac{\text{km}}{\text{s Mpc}}$ . In this case according to (8) we determine the average luminosity evolution

$$L(z) = R_0^2 10^{-7.44} z^{0.88}. \quad (10)$$

In the intermediate case of luminosity evolution  $L(z)=L z^\beta$ , where  $0 < \beta < 0.88$ , the distance is determined by expression

$$R_{\text{exp}} = L_0^{0.56} 10^{3.72} z^{\beta/2} \quad (11)$$

It is important to note that in the second case the luminosity evolution exceeds more than one order of magnitude both the theoretical estimations and recent data on possible evolution (Arimoto et al., 1992; Troitsky, 1993). The results of these investigations permit the luminosity evolution corresponding to  $\beta = 0.2-0.3$ , in this case  $R(z) = z^{0.56+\beta/2} = z^{2/3}$ .

The functional dependence of the distance to far objects  $R$  on  $z$  (9) and (11) greatly differ from the theoretical formula followed from the standard cosmology  $R = R_0 \frac{z}{z+1}$  (for  $q=1$ ), where event horizon  $R_0 = c/H_0$  equals 4000 Mpc, if Hubble constant is estimated as  $H_0 = 75 \frac{\text{km}}{\text{s Mpc}}$ . It is necessary to note that this theoretical relation is proved only in the red-shift interval  $z < 0.02$ , which contains about two per cent of distance described by  $R(z)$  function. It is quite naturally that a little part of any curve may be considered as a straight one. Thus, the observational result testifies against linear dependence of distance on  $z$  to nonlinear one. We discover that the dependence  $R(z) = z^{2/3}$  does not contradict to data which were used for the Hubble constant determination by way of linear approximation of

$m(z)$  dependence (for further details see, for example Rowan-Robertson, 1988).

Let us now define quantitative value of distance to objects. According to (9)  $D_0 = L_0^{1/2} 10^{3.72}$ . For  $L_0$  determination it is necessary to use the formula  $-2.5 \cdot \log(L_0) = M - 5$ . Thus,  $\log D_0 = -0.2$ ;  $M_0 = 4.72$ .  $M_0$  is the average absolute galaxy luminosity, within radius  $z = 0.02$ . The estimation  $20.5 < M < 21.5$  is reliable enough and for  $D_0$  we have

$$D_0 = (830 \pm 200) \text{ Mpc} \quad (12)$$

According to (9)  $D_0$  value have a physical sense of distance to galaxies with red-shift  $z=1$ . In standard cosmology this distance equals to 2000 Mpc ( $H_0 = 75 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$ ), it is approximately four times more than that given by our true general  $m(z)$  dependence.

These results lead to important conclusions. First, the absence or weakness of luminosity evolution solves the problem of very high quasar radiation power, which exceeds that of galaxies from one to three orders of magnitude according to standard cosmology estimations. In reality, radiation powers of a quasar and a galaxy are equal in average. This makes much weaker the problem of very high radiation density of quasars. Second, the decrease of distances to galaxies by 3-4 times remove completely the question on superluminal velocities of matter motion. In the paper (Cohen at all, 1988) the data on 32 objects are published, which have superluminal velocities according to the standard cosmology model. In our case the expansion velocity equals to  $D z \Omega$ , where  $\Omega$  is the obtained angle speed of relative motion in milliseconds per year. The calculation with  $D_0 = 830$  Mpc shows that 23 of 32 objects have underluminal velocities in the interval  $0.2c < v < c$  and for 9 sources this velocity does not exceed  $(2-2.5)c$ . Upper limit belongs to objects 3C-120, 3C-273, CTA-102. Now it is possible to make some important conclusions on Universe structure and evolution. First, the moment of the Universe beginning is absent, because red-shift  $z = \infty$  corresponds to an unlimited distant object. Second, the apparent luminosity evolution of galaxies and quasars is absent as well. That does not mean the absence of the evolution of every galaxy, but it means that the equilibrium between birthing and dying galaxies is established. According to general  $m(z)$  curve the brightest Sandage galaxies are flaming up in time, and weak galaxies are

fading. As a result the average luminosity does not vary. This conforms to the Universe infinity in time and space.

The main result is that the red-shift dependence on distance is not  $R - z$ , but  $R^2 - z$  for  $z < 1$ . That excludes both the Universe expansion theory of relativistic cosmology and the red - shift explanation by way of kinematic Doppler-effect. Thus, the result requires a new theoretical approach and comprehensive examination, the specification of  $m(z)$  dependence on  $z > 1$  is especially important, it will testify to or against the horizon existence. In conclusion we must note that our result is confirmed also by way of another cosmological test - the statistical dependence of a visual source angle dimension on the red-shift  $z$  (Troitsky, Aleshin, 1993).

It is necessary to note that quadratic dependence of the red-shift on distance is theoretically predicted in chronometrical cosmology developed by Segal (1976, 1979). However,

the spatial scale of the Universe in this theory is 50-100 times less than in standard cosmology. This does not confirmed by our results, the real redaction is 5-7 times. Finally, in paper (1989) he shows that for the little group of galaxies used the  $m(z)$  dependence did not correspond to Hubble law  $z - R$ . This observation better corresponds to  $z - R^2$  dependence.

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