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AN ALFVEN SWEEP MASER MODEL FOR PC 1 PEARLS: THEORY

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We review and extend a self-consistent model for the generation of Pc 1 pearl emissions, based on the nonlinear coupling between the magnetospheric and ionospheric resonators for Alfven waves. Following *Polyakov et al.* [1983]; *Belyaev et al.* [1987] we attribute the formation of pearls to the pulsating regime of the magnetospheric ion cyclotron maser with nonlinear selective mirrors. Such mirrors are formed by the conjugate ionospheres: their reflection coefficient has an oscillatory frequency dependence due to eigenmodes of the ionospheric Alfven resonator; nonlinear magnetosphere/ionosphere feedback is provided by the dependence of the value and frequency of the reflection maxima on the flux of energetic protons precipitated into the ionospheres in the course of Alfven wave generation in the magnetosphere. This feedback provides the formation of a single Alfven wave packet oscillating between the ionospheres; reflection coefficients evolve periodically in time, conjugate ionospheres being in opposite phases, so that a reflection maximum at a particular frequency occurs synchronously with the arrival of the corresponding part of the wave packet.

We present an analysis of a mathematical model for the system described, called Alfven sweep maser (ASM), to confirm the above qualitative formulation. We also discuss the ability of this model to explain observational characteristics of Pc 1 pearls, such as their morningside predominance, correlation with low magnetic activity, spatio-temporal and spectral patterns.

1 Introduction

Pc 1 pearl-type pulsations are one of the most regular phenomena in the magnetosphere. Main features of Pc 1 pearls have been known for many years [see e.g. monographs by *Guglielmi and Troitskaya*, 1973; *Nishida*, 1978]. These features are: (1) Periodicity with typical values of a period $T \sim 10^2$ s; (2) Increasing frequency inside a wave packet; (3) Anti-correlation in conjugate ground-based observation points; (4) Morningside predominance; (5) Correlation with late recovery phase of geomagnetic storm.

In theory, we know the basic mechanism of Pc 1 generation, which is cyclotron instability of energetic protons with anisotropic velocity distribution. Predominance of pearl observations during recovery phase of a storm is attributed to radial broadening of plasmapause during recovery stage and associated encounter of previously injected outer ring current protons in region of enhanced plasma density, that provides conditions for cyclotron instability [*Nishida*, 1978]. Temporal characteristics of pearls are most easily connected with a short Alfven wave packet oscillating between conjugate ionospheres [*Tepley*, 1964; *Jacobs and Watanabe*, 1964]. Some spectrum features of Pc 1 pearls are understood on the basis of a linear model suggested by *Gendrin et al.* [1971], who investigated the evolution of a packet with account of amplifying and dispersive properties of magnetospheric plasma. This theory related pearl characteristics to a set of parameters of magnetospheric plasma, such as background plasma distribution and temporal evolution of the instability growth rate. However, processes that lead to a certain evolution of growth rate are still not well understood.

There are several mechanisms which in principle may contribute to pearl formation. *Olson and Lee* [1983] discussed a possibility to relate Pc 1 pearls with modulation of ion cyclotron instability by hydromagnetic waves. Observational support of this hypothesis is not evident, in particular, presence of compressional pulsations of substantial power, correlated with Pc 1, has not been reported. There is also an idea to explain temporal modulation of growth rate by existence of a phase-correlated bunch of energetic protons that provides periodic generation of Pc 1 emissions [*Jacobs and Watanabe*, 1963; *Erlandson et al.*, 1992]. This explanation requires an external source of highly correlated proton population which seems a rare occasions during storm recovery; observations of bunched helium ions, reported by *Mauk and Mc Pherron* [1980], are attributed to energization of those minority ions through cyclotron interaction with proton-generated cyclotron waves, but such a process is not possible for protons themselves.

The mechanisms mentioned are based on an external driver for Pc 1 formation. There are also self-consistent processes which may lead to pulsating character of wave generation and to formation of fine spectral structure. (1) One of them is related with the background plasma nonlinearity leading to wave self-modulation [*Petviashvili*, 1979]. (2) The second mechanism is connected with side-band instability of a quasi-monochromatic ion-cyclotron wave due to trapping of energetic particles by the wave field [*Bud'ko et al.*, 1972]. (3) The third mechanism is fast modulation of the instability growth rate due to quasi-linear pitch-angle diffusion. Evolution of proton cyclotron instability with account of wave dispersion and particle quasi-linear diffusion, in absence of energetic particle source, was considered by *Cocke and Cornwall* [1967]; under the action of constant supply of energetic particles quasi-linear diffusion may lead to a spike-like regime of wave generation [*Bespalov*, 1982, 1984; *Demekhov and Trakhtengerts*, 1986, 1994]. Including the influence of energetic particle distribution on wave dispersion properties into consideration and taking into account finite time of wave propagation between hemispheres, one obtains pulsation regimes with frequency modulation and characteristic period equal to wave group period T_g [*Bespalov*, 1984]. (4) The fourth mechanism is connected with the nonlinearity and selective properties of ionospheric mirrors. In the case of Alfven waves, whose wavelength is comparable with ionosphere height, the influence of ionospheric Alfven resonator (IAR) eigenmodes on the wave reflection is very important, providing an oscillatory frequency dependence of the reflection coefficient [*Polyakov et al.*, 1983; *Lysak*, 1990, 1993]. The nonlinear modification of wave reflection coefficient from the ionosphere is caused by precipitating energetic protons; e.g. the influence of precipitated energetic protons, resulted from pearl generation, on ionospheric electron density, was shown by *Mende et al.* [1980]. They have found the correlation, with the appropriate time lag, between a pearl packet, a burst of proton precipitation into the conjugate ionosphere, and corresponding rise of ionospheric electron density. The mentioned properties of ionosphere reflection coefficient may result in the formation of fine time and spectral structure of Pc-1 pulsations [*Polyakov et al.*, 1983; *Belyaev et al.*, 1984, 1985, 1987].

All these processes may probably influence the formation of Pc 1 pearls but none of them is now considered seriously when discussing experimental results in this area. In the present paper we would like to review the self-consistent model connected with nonlinearity of ionosphere reflection, suggested by *Polyakov et al.* [1983]; *Belyaev et al.* [1984, 1985, 1987] and called Alfven sweep maser (ASM). We also discuss the relation between this model and known experimental data. As we well show below, it provides a consistent explanation for all main characteristics of Pc 1 pearls that were enumerated above. In particular, the ASM mechanism, unlike other models mentioned, explains anticorrelation in time between Pc 1 registrations at conjugate ionospheres [*Saito*, 1969]

In the following we will concentrate on the discussion of the principal features of the ASM model, so we will not take into account several processes that may also be of a certain importance. In particular, we will discuss only the quasi-linear theory, and not the nonlinear trapping effects. We think this is appropriate for low disturbance levels, although at highest observed amplitudes of Pc 1 pearls nonlinear bunching may become significant [*Bud'ko et al.*, 1972]. Also we will not consider effects of heavy ions on the cyclotron instability, assuming some given frequency dependence of the growth rate. We note that according to observations [*Erlandson et al.*, 1992; *Mursula et al.*, 1994], He^+ ions may have a substantial influence on the frequency band of pearls. In this paper we discuss the frequency ranges outside of a gap in cyclotron amplification, situated near the He^+ gyrofrequency.

2 Summary of Alfven Sweep Maser Model

2.1 Basic equations

2.1.1 Equations for cyclotron instability

The ASM model is currently based on self-consistent quasi-linear equations for cyclotron resonant interaction of energetic protons with electromagnetic waves, propagating parallel to the external magnetic field $\vec{B}$. In this paper we will consider the case

$$\omega \ll \Omega_B \tag{1}$$

where Ω_B is the proton cyclotron frequency. From the cyclotron resonance condition

$$\omega - \Omega_{BL} = k_{\parallel} v_{\parallel} \tag{2}$$

combined with an approximate dispersion relation for low-frequency Alfven waves,

$$k \simeq \frac{\omega}{c} n_A \qquad n_A = \frac{\Omega_p}{\Omega_B} \tag{3}$$

we can see that the inequality $T_g \gg T_b$ holds where T_g is the group period of wave packet bouncing between conjugate hemispheres, and T_b is the proton bounce time. So we can distinguish processes for which only group delay effects are substantial and bounce resonance effects play no role. For studying such processes we can use the bounce-averaged kinetic equation for protons, while spatial (along $\vec{B}$) inhomogeneity should be taken into account in wave energy transfer equation.

The inequality (1) means also that pitch-angle diffusion dominates over the energy diffusion. We thus have the following set of equations:

$$\frac{\partial f}{\partial t} = \frac{1}{T_b} \frac{\partial}{\partial \eta} \eta D \frac{\partial f}{\partial \eta} + J(t, \eta, v) \tag{4}$$

$$\frac{\partial \mathcal{E}^{\pm}}{\partial t} \pm V_g \frac{\partial \mathcal{E}^{\pm}}{\partial z} = (\gamma - \nu) \mathcal{E}^{\pm} + a_{\omega} \tag{5}$$

where $f(t, \eta, v)$ is proton distribution function, $J(t, \eta, v)$ is the particle source, $\eta = \sin^2 \theta_L$, θ is the particle pitch-angle, index "L" denotes values in the equatorial plane,

$$T^{-1} = \begin{cases} 2/T_b, & \eta \le \eta_c, \\ 0, & \eta > \eta_c, \end{cases} \tag{6}$$

$\eta_c = \sigma^{-1}$ is the loss cone pitch angle, σ is the mirror ratio. In the equation (5) $\mathcal{E}^{\pm}(t, \omega, z)$ is the spectral energy density of Alfven waves, V_g is the wave group velocity, ν is wave damping rate, a_ω is the external wave source including spontaneous emissivity; signs $\pm$ refer to waves in $\pm\vec{z}_0$ directions.

The pitch-angle diffusion coefficient D in (4) is determined by the integration of wave spectral energy density over ω and over the length of a magnetic mirror [*Bespalov and Trakhtengerts*, 1980]:

$$D(t, \eta, v) = \frac{8\pi^2 e^2}{m^2 c^3 v^3} \cdot \oint \frac{\eta b(z) dz}{\sqrt{1 - \eta b(z)}} \int \left(\frac{\Omega_B}{\omega} - \eta \right)^2 \mathcal{E}_{\omega}^{\Sigma} \times$$
$$\times v_{ph} V_g \, \delta \left(\omega - \Omega_B + kv\sqrt{1 - \eta b(z)} \right) d\omega \tag{7}$$

where $b(z) = B(z)/B_L$, $v_{ph} = \omega/k$ is wave phase velocity, $\mathcal{E}_\omega = \mathcal{E}_\omega^+ + \mathcal{E}_\omega^-$.

The wave growth rate γ in (5) is calculated from the equation

$$\gamma(t,\omega,z) = \frac{4\pi^3 e^2}{mc^2} v_{ph}\, v_g\, b(z) \int_0^1 \frac{d\eta}{\sqrt{1-\eta b(z)}} \int v^2\, dv \times \tag{8}$$
$$\times \left[\left(\frac{\Omega_B}{\omega} - 1\right)\eta\frac{\partial f}{\partial \eta} - f\right] \delta\left(\omega - \Omega_B + kv\sqrt{1-\eta b(z)}\right).$$

The formula (8) was obtained integrating the known expression for wave growth rate [*Bespalov and Trakhtengerts*, 1980] by parts to exclude the derivative $\partial f/\partial v$. It is worth noting that the last term in square brackets determines the upper frequency of unstable waves, $\omega_{max} = A\omega_{BL}/(A+1)$, where A is the anisotropy index; we may omit this term if for some reason, e.g. because of frequency dependence of reflection coefficient, waves are excited at $\omega \ll \omega_{max}$.

The precipitating particle fluxes $S^\pm$ in the equation (5) are calculated as

$$S^\pm = \frac{\pi\sigma}{2}\int_0^\infty D^\pm \left.\frac{\partial f}{\partial \eta}\right|_{\eta=\eta_c} v^3\, dv, \tag{9}$$

$D^\pm$ being determined by (7) but with $\mathcal{E}_\omega$ replaced by $\mathcal{E}^\pm$.

Equation (4) should be solved with the following boundary conditions:

$$f|_{\eta=\eta_c} = 0 \tag{10}$$

$$\left. D\frac{\partial f}{\partial \eta}\right|_{\eta=\eta_m=1} = 0. \tag{11}$$

The boundary condition (10) corresponds to the so-called weak diffusion limit [*Kennel*, 1969; *Bespalov and Trakhtengerts*, 1980] which corresponds to sufficiently low level of magnetic disturbance.

The boundary conditions for the wave transfer equation (5) have the form

$$\begin{aligned} \mathcal{E}^-(t,\omega,l) &= R_1\mathcal{E}^+(t,\omega,l), \\ \mathcal{E}^+(t,\omega,-l) &= R_2\mathcal{E}^-(t,\omega,-l), \end{aligned} \tag{12}$$

where $R_{1,2}$ are reflection coefficients from conjugate ionospheres, and $z = \pm l$ are coordinates of ionospheres.

2.1.2 A reflection coefficient from the ionosphere

The role of the ionosphere in wave-particle interaction is exhibited in the boundary conditions for the equation (5). Those conditions have the form (12). Ionospheric reflection coefficient $R_{1,2}$ in Alfven frequency band has oscillatory frequency dependence, which is determined by the influence of ionospheric Alfven resonator (IAR) [*Polyakov*, 1976]. Analytical calculations of the IAR properties were done by *Polyakov and Rapoport* [1981]; *Trakhtengerts and Feldstein* [1981]; *Polyakov et al.* [1983]; *Lysak* [1988]; also numerical investigation was undertaken with account of real altitudinal profiles of electron number density [*Ostapenko and Polyakov*, 1990; *Lysak*, 1993]. The model altitudinal profile of the Alfven wave refractive index $n(h)$ in the ionosphere was used to determine the spectrum of the IAR eigenmodes analytically:

$$n^2 = \begin{cases} n_A^2\left(\varepsilon^2 + \exp\left\{-2\frac{h-h_2}{L}\right\}\right), & z \geq h_2, \\ n_A^2(1+\varepsilon^2), & h_1 \leq z \leq h_2, \end{cases} \tag{13}$$

where n_A is the refractive index at the F-layer maximum, $\varepsilon \ll 1$. The lower ionosphere region $h < h_1$ in those calculations was characterized by the impedance

$$Z_i = -n_A^{-1}\left[\frac{\Sigma_P}{\Sigma_W} - ik_A h_0\left(\frac{\Sigma_H}{\Sigma_W}\right)^2\right]^{-1}, \tag{14}$$

where Σ_P and Σ_H are height-integrated Pedersen and Hall conductivities of the ionosphere, $\Sigma_W = cn_A/4\pi$ is the wave conductivity of the F-layer, h_0 is the height of the vacuum gap between the Earth and the ionosphere, $k_A = k_0 n_A = (\omega/c)n_A$.

For this model of $n(h)$ IAR eigenmodes are expressed in terms of Bessel functions. With the inequalities

$$\varphi_0 \equiv n_A k_0 L \gg 1, \qquad a \equiv \frac{1}{2}\pi\varepsilon\varphi_0 \ll 1 \tag{15}$$

which hold in the Alfven wave frequency range, the following formula for the reflection coefficient was obtained [*Polyakov et al.*, 1983]:

$$|R|^2 = \frac{[-Ba\varphi_0 + (a\varphi_0 - A)\tan\varphi_1]^2 + [-Aa\varphi_0 + B\tan\varphi_1 + 1]^2}{[-Ba\varphi_0 + (a\varphi_0 + A)\tan\varphi_1]^2 + [Aa\varphi_0 + B\tan\varphi_1 + 1]^2}, \tag{16}$$

where $\varphi_1 = \varphi_0 - \frac{\pi}{4}$ $A = -n_A \mathrm{Re} Z_i$, $B = -n_A \mathrm{Jm} Z_i$.

The value of $|R|^2$ has an oscillatory dependence on ω [*Polyakov et al.*, 1983]. The position, amplitude and width of its maxima is a function of ionospheric electron number density, which is strongly influenced by energetic proton precipitation from the magnetosphere. This influence can be taken into account by writing down ionization balance equations

$$dn_i^{\pm}/dt = I + QS^{\pm} - \chi(n_i^{\pm})^2, \tag{17}$$

where $n_i^{\pm}$ are the height-integrated ionospheric number densities, χ is the recombination coefficient, I is quiet-time ionization source, $S^{\pm}$ are the energetic particle precipitation fluxes, which are calculated according to the formula (9).

2.2 Simplifications of basic equations for ASM

The formulae for the diffusion coefficient D (7) and growth rate γ (8) are rather complicated, so analytical and numerical investigation of the full model would be very difficult. It is also important that the equation set given in the previous sections includes two different nonlinear effects influencing the spectrum dynamics, such as growth rate modification due to quasi-linear diffusion and nonlinearity of ionospheric mirrors. Each of these effects is not simple, so it would be better to study them separately. In particular, to make the influence of the IAR on the fine structure formation more clear, we can consider the situation when quasi-linear effects do not change wave spectrum substantially. It is usually true if (1) weak pitch-angle diffusion regime takes place, (2) $\omega \ll \omega_B$ and (3) the cyclotron amplification is not large, $\Gamma = \oint(\gamma/V_g)\,dz < 1$.

In this case we can adopt a simplified approach to the description of particle dynamics in radiation belts which yield more straightforward calculation of the diffusion coefficient and Alfven wave growth rate. The simplest form of quasi-linear equations is the so-called two-level, or balance approximation which is obtained under the assumption of a fixed pitch-angle distribution function of particles. This is correct when the distribution function coincides with a stable eigenfunction $\mathcal{F}(\eta)$ of quasi-linear diffusion operator:

$$\frac{1}{T_b}\frac{\partial}{\partial\eta}D\frac{\partial\mathcal{F}}{\partial\eta} = -\mathcal{F}/T_l \tag{18}$$

In the balance approximation, the kinetic equation (4) is replaced by an ordinary differential equation for energetic particle content in the magnetosphere:

$$\frac{dN}{dt} = -S^{\Sigma} + I_0 \tag{19}$$

where $N = \pi\sigma \int fT_b v^3 \, dv \, d\eta$ is the number of particles in a flux tube with the unit cross-section at ionosphere, and precipitating particle flux $S^\Sigma = S^+ + S^-$ can be represented, according to (7) and (9), as

$$S^\Sigma = \int_0^\infty \int_{-l}^{l} \phi_1(z,\omega)\mathcal{E}_\omega^\Sigma \, dz \, d\omega \tag{20}$$

where $\phi_1(z,\omega)$ is found from (7) and (9) by substituting the eigenfunction $\mathcal{F}$ (18) instead of f. The growth rate γ in the wave transfer equation (5) should also be calculated with the preset distribution function $f = \mathcal{F}$ and may be written as follows:

$$\gamma = \frac{N}{N_c}\Omega_{BL} \, \phi_2(z,\omega) \tag{21}$$

where the index "c" refers to the background plasma. the dependence of $\phi_2(z)$ is essentially the same as of the function $\phi_1(z)$ in (20). For analytical investigations discussed in the following sections, these functions will be, where necessary, approximated by a rectangle:

$$\phi_{1,2}(z,\omega) = (2d)^{-1}\varphi_{1,2}(\omega)[H(z+d) - H(z-d)] \tag{22}$$

$H(x)$ is the Heavyside step function.

These simplifications make it possible to obtain explicit solutions for stability analysis of the self-consistent set of ASM equations.

3 ASM Dynamics as Related to Characteristics of Pc 1 Pearls

3.1 Stationary solution of the ASM equations

The ASM equations (5), (17), and (19), together with the boundary conditions (12), have a stationary solution, corresponding to a dynamical balance between the energy supply via energetic proton source and energy loss due to particle precipitation into the ionosphere and wave damping. We will consider the formation of Pc 1 pearls as a consequence of an instability of that stationary solution, so it is useful to write it down explicitly.

Integrating the wave equation (5) and taking the boundary conditions (12) into account, we obtain the following relation:

$$\mathcal{E}_0^\pm(z,\omega) = \frac{a_\omega}{1 - R_- R_+ \exp(4\Gamma_0)} \, \psi_\mathcal{E}^\pm(\gamma_0, z) \tag{23}$$

where

$$\pm\psi_{\mathcal{E}}^{\pm}(\gamma_0,z) = \int_{\mp l}^{z} e^{\mp G_0(z,z')}\frac{dz'}{V_g} + R_-R_+e^{4\Gamma_0}\int_{z}^{\pm l} e^{\mp G_0(z,z')}\frac{dz'}{V_g} + R_{\mp}e^{\pm G_0(\mp l,z)}\int_{\pm l}^{\mp l} e^{\pm G_0(\mp l,z')}\frac{dz'}{V_g} \tag{24}$$

where $G_0(z_1,z_2) = \int_{z_1}^{z_2}\gamma_0\,dz/V_g$, $\gamma_0 = \gamma(N_0)$ is the stationary local growth rate, $\Gamma_0 \equiv G_0(0,l)$ is the stationary wave intensity gain on the pass from the equator to the ionosphere.

We see that $\mathcal{E}_0^{\pm}$ depend on stationary values of $\Gamma_0 \propto N_0$, and $R_{\pm}(n_i^{\pm})$. These may be obtained using the equations (17), (19), and (16), (20). They are transcendent equations including the integration over ω and z. Finding $n_i^{\pm}$ is greatly simplified for the case of symmetric ionospheres, then we have $S_0^+ = S_0^- = S_0$; from (19) we obtain

$$S_0 = I_0/2 \tag{25}$$

$$(n_i^+)^2 = (n_i^-)^2 = \chi^{-1}(I + QI_0/2) \tag{26}$$

Thus we know ionosphere parameters and may calculate the reflection coefficients. The flux S_0 depends on $\mathcal{E}$, so we must solve equations (23) and (25), to determine wave amplitude and stationary content of energetic protons in the magnetosphere.

At $a_\omega \to 0$ the stationary frequency spectrum $\mathcal{E}_0$ formally approaches δ-function at the frequency ω_0, corresponding to the maximum of total amplification:

$$\begin{aligned}\mathcal{E}_0 &\to E_0(z)\delta(\omega-\omega_0)\\ \omega_0 &= \{\omega|\quad \partial[R_+R_-e^{4\Gamma_0}]/\partial\omega = 0\}\end{aligned} \tag{27}$$

In this case the stationary value of N is determined by the relation

$$\left(R_+R_-e^{4\Gamma_0}\right)_{\max} = 1 \tag{28}$$

At small a_ω the stationary wave spectrum has a narrow maximum at $\omega = \omega_0$, where $1 - R_+R_-e^{4\Gamma_0} \ll 1$. So the physical sense of the equation (27) is that the stationary spectrum may be narrow as compared with other frequency scales, such as the amplification line width and the scale of $R_{\pm}(\omega)$ variation; at the same time it may remain inside the applicability limits of the quasilinear theory. The case of relatively narrow stationary wave spectrum is important because all the integrations over ω are then greatly simplified, and rather simple and physically clear analysis of the stationary state stability is possible.

3.2 Pulsating regime of Pc 1 generation in adiabatic approximation

Because the equations for ASM are rather complicated, we investigate their features step by step. First consider the simplest model taking into account only nonlinearity of ionospheric mirrors on the action of Alfven ion cyclotron maser [*Polyakov et al.*, 1983]. For that we will use the balance equation (19) for particles, and average the wave equation (5) over group period T_g, to obtain the "adiabatic" approximation

$$\frac{\partial \mathcal{E}_\omega}{\partial t} = [\hat{\gamma}(\omega) - \nu]\,\mathcal{E}_\omega + a_\omega \tag{29}$$

where $\hat{\gamma} = T_g^{-1}\oint(\gamma/V_g)\,dz = \Omega_{BL}(N/N_c)\,(d/l)\,\varphi_2(\omega)$ (see (21) and (22)), $\nu = T_g^{-1}|\ln R_1 R_2|$.

We will review the results for adiabatic approximation very briefly, to understand the onset of undamped Pc 1 pulsations via the ionosphere nonlinearity, and to be able to compare these results with a more elaborate model taking group delay effects into account. For small a_ω, stationary wave spectrum is narrow, $\omega \approx \omega_0$, so let us integrate (29) over ω, to get

$$\frac{\partial E}{\partial t} = (\gamma_m - \nu_0)E + a \tag{30}$$

$E = \int \mathcal{E}_\omega\, d\omega$, $\gamma_m = \hat{\gamma}(\omega_0)$, $\nu_0 = \nu(\omega_0)$. In this case the precipitating particle flux is written as [*Bespalov and Trakhtengerts*, 1980]: $S = (\Omega_{BL}/\omega)\gamma_m N E/W_0$, W_0 is the energy of protons. During the cyclotron instability, parameters of ionosphere are changed under the influence of precipitated energetic protons. According to (16), this causes variation of the reflection coefficient $R(\omega)$. Near the steady state we may take these variations into account by expanding γ_m and ν_0 over $n_i - n_0$:

$$\nu_0 = \nu_{00} + \frac{\partial \nu}{\partial n_i}(n_i - n_0); \qquad \gamma_m = \gamma_{m0} + \frac{\partial \gamma_m}{\partial n_i}(n_i - n_0) \tag{31}$$

The analysis of stability of the steady state of the equation set (17),(19), and (30) is straightforward; it gives the following criterion for transition of ASM from stationary to spike-like generation:

$$\frac{\nu_0 \tau_r Q N_0}{1 + \tau_r^2 \Omega_R^2}\frac{\partial}{\partial n_i}\ln(\gamma_m \nu_0) > 1 \tag{32}$$

where $\tau_r^{-1} = 2\chi n_0$, $\Omega_R^2 = \gamma_{m0}/T_0$ is the frequency of damped relaxation oscillations in absence of ionosphere nonlinearity [*Bespalov and Trakhtengerts*, From equations (23) and (24) we can obtain that for small a_ω

$$\begin{gathered}\left(R_0 e^{2\Gamma_0}\right)_{\omega=\omega_0} \simeq 1-\Delta_a \\ \Delta_a \simeq \left(\frac{a_\omega T_g}{\mathcal{E}_{0L}} \frac{\sinh\Gamma_0}{\Gamma_0}\right)_{\omega=\omega_0} \ll 1, \qquad T_g=\frac{2l}{V_g}\end{gathered} \tag{42}$$

Thus equation (41) is satisfied if

$$\Lambda=\Lambda_n \simeq i\Omega_n+\frac{\Delta_n}{2}, \qquad \Omega_n=\pi(n+\frac{1}{2}) \qquad n=0,1,2,\ldots \tag{43}$$

$(\Lambda \equiv \lambda l/V_g)$. Substituting (43) into (39), we find

$$\Delta=\Delta_n+\Delta_a=Q\left(g_{\omega\lambda}\frac{\partial R}{\partial n_i}\frac{E_0 e^{\Gamma_0+\Gamma_\lambda}}{\lambda+\nu_r}\right)_{\omega=\omega_0} \tag{44}$$

where we can use the zero-order approximation, $\Lambda_n=i\Omega_n$, at the right-hand side, to obtain a first-order approximation for Δ. Thus equations (43) and (44) give an explicit complex solution of the characteristic equation (39) for antisymmetric modes. This formula is valid until the resulting Δ is small, $|\Delta| \ll 1$.

The term $g_{\omega\lambda}$ (see the Appendix) depends on z-profile of $\phi_1(z,\omega)$; using the rectangle approximation (22), we get

$$g_{\omega\lambda}=\phi_{10}\cdot 2l\cdot\frac{\sinh\Gamma_\lambda}{\Gamma_\lambda} \tag{45}$$

Taking into account that, according to (20), (22)–(25), $S_0=I_0/2=2l\phi_{10}E_0$, we get the following final expression for Δ:

$$\mathrm{Re}\Delta_n+\Delta_a = -\mathcal{R}\frac{\Lambda_r\Gamma_0+\Omega_n^2}{(\Lambda_r^2+\Omega_n^2)(\Gamma_0^2+\Omega_n^2)} \tag{46a}$$

$$\mathrm{Jm}\Delta_n = -\mathcal{R}\frac{\Omega_n(\Lambda_r-\Gamma_0)}{(\Lambda_r^2+\Omega_n^2)(\Gamma_0^2+\Omega_n^2)} \tag{46b}$$

$$\Lambda_r = \nu_r l/V_g \tag{46c}$$

$$\mathcal{R}=\frac{1}{8}QI_0\left(\frac{\partial R}{\partial n_i}\frac{l}{V_g}\frac{(1+R)|\ln R|}{R(1-R)}\right)_{\omega=\omega_0} \tag{47}$$

If we neglect Δ_a on the left-hand side, then the sign of $\mathrm{Re}\lambda=\mathrm{Re}(\Delta_n)V_g/l$ is determined by the sign of ionosphere reflection nonlinearity, $\partial R/\partial n_i$. The stationary state is unstable for antisymmetric disturbances if $\partial R/\partial n_i < 0$. Let us explain this fact by analyzing equation (60). For purely antisymmetric disturbances $e_L^{(s)} \equiv \frac{1}{2}(e_L^+ + e_L^-) = 0$, so from (58) and (65) one obtains that $N_\lambda = 0$. Thus we have

$$e_\lambda^\pm(z) \simeq \pm\frac{1}{2}e_{\lambda L}e^{\pm G_\lambda(z)}, \qquad e_\lambda(z) = e_{\lambda L}\sinh(\Gamma_\lambda z/l) \tag{48}$$

where $e_{\lambda L} = e_{\lambda\,L}^+ = -e_{\lambda\,L}^-$. Substituting (48) into (62), and using (22), we get

$$\begin{aligned} n_\lambda^\pm &= \mp Ql\phi_{10}e_{\lambda L}\frac{\sinh\Gamma_\lambda}{\Gamma_\lambda(\lambda+\nu_r)} \\ &= 2Ql\phi_{10}\tfrac{l}{V_s}\,e_\lambda^\pm(\pm l)\,\frac{1+R}{(\Gamma_0-i\Omega_n)(\Lambda_r+i\Omega_n)} \end{aligned} \tag{49}$$

Analyzing the phase of the complex denominator on the right-hand side, we see that the maximum of an incident wave always corresponds to negative variation of n_i (at $\Lambda_r = \Gamma_0$ they are in opposite phases). So at $\partial R/\partial n_i < 0$ an antisymmetric wave disturbance coming at ionosphere level always meets an increase of the reflection coefficient.

For the symmetric disturbance mode, $e^{(s)}$, the simplified dispersion relation (39) after all transformations acquires the following form [*Belyaev et al.*, 1984]:

$$(\lambda + T_0^{-1})\sinh\Lambda \approx \frac{\mathcal{R}\lambda}{\Gamma_\lambda+\Lambda_r}\,\frac{\sinh(\Gamma_\lambda)}{\Gamma_\lambda} \tag{50}$$

(here an additional assumption $\Gamma_0 < 1$ was used). One may see that the solutions to this dispersion relation are imaginary at $|\Gamma_\lambda| \gtrsim 1$, i.e. symmetric disturbances with spatial scales $\lambda \lesssim 2l$ are stable for $\mathcal{R} \ll 1$. For the case $|\Gamma_\lambda| \ll 1$ we obtain the adiabatic approximation discussed in the section 3.2, which has unstable solutions if the nonlinearity of ionosphere reflection exceeds a rather high critical value, $\mathcal{R} > \mathcal{R}_*$.

If the parameters of conjugate ionospheres are not the same, then the antisymmetric and symmetric modes are not independent, so the analysis is more complicated, and we will not consider it in details. An approximate solution, obtained by *Belyaev et al.* [1985] for the case of weak coupling between the modes, showed that the instability of the stationary state of ASM is possible even if ionosphere nonlinearities are of opposite signs, but in this case it is weaker than for similar ionospheres. Symmetric disturbance modes are damped, except the adiabatic mode, which is not sensible to the difference between ionospheres, and is determined by parameters of total reflection coefficient, $R = R_+R_-$ (see section 3.2).

1976; *Davidson*, 1979], $T_0 = N_0/S_0$ is the particle lifetime at the stationary state.

Thus in adiabatic approximation ionosphere nonlinearity may cause a spike-like regime of Alfven wave generation if it provides a growth of the total gain during the initial stage of the instability, which exceeds the damping rate of relaxation oscillations. As shown by *Polyakov et al.* [1983], most favorable conditions for (32) to be satisfied exist at the morningside during quiet periods (see also section 4); however the inequality (32) turns out to be rather restrictive, so from the adiabatic model we get rather high threshold for formation of Pc 1 pearl emissions via the ASM mechanism.

3.3 Spatial structure of an unstable wave packet

In this section we will show, on the basis of a linear stability analysis, that the combination of finite wave propagation time between ionospheres and nonlinearity of ionospheric mirrors gives preference to the formation of one wave packet oscillating in the magnetosphere. In the following we will use the balance approximation, so variation of the distribution function includes only variation of the number density, according to the equation (19). Linearize the initial equations and boundary conditions near the stationary solution, discussed in the section 3.1 and introduce the Laplace transformation:

$$u(t) = u_0 + u_\sim, \quad |u_\sim| \ll u_0 \quad u(t) = \int_{-i\infty}^{i\infty} \exp(\lambda t) u_\lambda \, d\lambda \tag{33}$$

where u stands for N, $\mathcal{E}^\pm$ or $n_i^\pm$.

It is convenient to introduce the symmetric and antisymmetric disturbance modes according to the transformation

$$e_\lambda^{(s,a)} = e_\lambda^+ \pm e_\lambda^- \tag{34}$$

One can easily understand that the antisymmetric mode corresponds to the tendency of formation of one oscillating wave packet, whereas the symmetric mode corresponds to at least two packets placed symmetrically relative to the equator.

The most significant details of calculations for the linearized ASM set of equation are given in the Appendix. Consider first the simplest case when the hemispheres are symmetric. Then the linearized ASM equations are split into two independent sets for symmetric (e^s) and asymmetric (e^a)

modes:

$$e_{\lambda L}^{(a,s)} = \frac{K_{\lambda\omega}^{(a,s)}}{1 \pm R e^{2\Gamma_\lambda}} \int g_{\omega\lambda} e_{\lambda L}^{(a,s)}\, d\omega \qquad (35)$$

where $\Gamma_\lambda = \int_0^l (\gamma_0 - \lambda)\, dz/V_g$,

$$K_{\lambda\omega}^{a} = Q \frac{\partial R}{\partial n_i} \frac{\mathcal{E}_{0L} + A_0}{\lambda + \nu_r} e^{\Gamma_0 + \Gamma_\lambda} \qquad (36)$$

$$K_{\lambda\omega}^{s} = \left\{ \lambda K_{\lambda\omega}^{a} + \frac{(g_+ + g_-)\mathcal{E}_{0L} + A_+ - A_-}{N_c} \right\} \times \left[\lambda + T_0^{-1} - P_\lambda\right]^{-1} \qquad (37)$$

$P_\lambda = N_c^{-1} \int_0^\infty h_{\omega\lambda}\, d\omega$, the expressions for all incoming values are given in the Appendix.

Multiplying the equations (35) by $g_{\omega\lambda}$ and integrating by ω, we will get the characteristic equations for finding λ:

$$\int \frac{g_{\omega\lambda} K_{\lambda\omega}^{(a,s)}}{1 \pm R_0 \exp(2\Gamma_\lambda)}\, d\omega = 1 \qquad (38)$$

From equations (35)–(37) we see that the solution for wave disturbance includes a part, determined by a stationary wave spectrum, $\mathcal{E}_{0\omega}$, and an addition due to external wave sources a_ω. At small a_ω the frequency spectrum of a stationary solution $\mathcal{E}_{0\omega}$ is relatively narrow (see section 3.1), so we may approximately perform an integration in (38) and obtain

$$\left(\frac{\mathcal{K}_{\lambda\omega}^{(a,s)}}{1 \pm R_0 \exp(2\Gamma_\lambda)} \right)_{\omega=\omega_0} \cdot E_0 + A_\delta(\lambda) = 1 \qquad (39)$$

where $E_0 = \int \mathcal{E}_{0L}\, d\omega$, $A_\delta(\lambda)$ is a small addition due to a_ω, which is neglected below, and

$$\mathcal{K}_{\lambda\omega}^{(a)} = Q \frac{\partial R}{\partial n_i} \frac{g_{\omega\lambda} e^{\Gamma_0 + \Gamma_\lambda}}{\lambda + \nu_r} \qquad (40a)$$

$$\mathcal{K}_{\lambda\omega}^{(s)} = \frac{\lambda \mathcal{K}_{\lambda\omega}^{(a)} + (g_+ + g_-)/N_c}{\lambda + T_0^{-1} - P_\lambda} \qquad (40b)$$

Let us consider the asymmetric modes. If the nonlinearity of the ionosphere reflection is small enough, so that $\mathcal{K}_{\lambda\omega}^{(a)} E_0 \ll 1$, then the solution of equation (39) is near the pole of its left-hand side:

$$1 + R_0 e^{2\Gamma_\lambda}\big|_{\omega=\omega_0} = \Delta \ll 1 \qquad (41)$$

To summarize, in this section we have shown that the growth of antisymmetric wave disturbances is favored at $\frac{\partial R}{\partial n_i} < 0$, and that the fastest growing mode is that with $n = 0$, which corresponds to anti-correlated oscillations of wave intensity at conjugate ionospheres. This is confirmed by an illustration shown in Figure 1, where we plotted profiles of this mode ($\lambda = \lambda_0$) for different time moments. By definition (33) $e(z,t) = e_{\lambda_0}(z)e^{\lambda_0 t}$, where we use the formula (48) for $e_{\lambda_0}(z)$. Plotted profiles were multiplied by $e^{(\mathrm{Re}\lambda t)}$, to exclude growth of the disturbance amplitude. The upper panel shows deviations of total wave intensities, and the lower panel presents deviations of intensities of downward and upward waves, e_λ^+ and e_λ^-, separately at one time moment. Negative signs for $e_\lambda^\pm$ obviously correspond to the decrease of the amplitude relative to the stationary solution. These plots are obtained for $\mathrm{Re}\Delta_0 = 0.114$, $\mathrm{Jm}\Delta_0 \simeq 0.02$; these values, in particular, correspond to the following choice of the dimensional parameters: $I_0 = 3 \cdot 10^4$ cm^{-2} s^{-1}, $\chi = 10^{-6}$ cm^3 s^{-1}, $Q = 10^{-2}$ cm^{-1}, $l/V_g = 25$ s (this corresponds to total period $T = 2T_g = 100$ s), $\frac{\partial R}{\partial n_i} = -2 \cdot 10^{-5}$ cm^3.

3.4 Dynamics of Alfven wave spectrum in ASM

In the previous discussion we considered small deviations from a stationary state of Alfven sweep maser and determined the conditions when this steady state may be unstable due to nonlinear change of ionosphere reflection coefficient. Using results presented, it is possible to find the evolution of a wave packet at small deviations from the stationary state. In general, it may be done by expanding the disturbance profile over discrete series of eigenfunctions $e_\lambda^{(a,s)}$, for which eigenvalues have been found above. It is clear, however that in the case of unstable stationary state the mode with the highest growth rate will dominate after some time. So let us follow the evolution of the antisymmetric disturbance with $n = 0$ (see equation (46)). It is easily to find that if $e^{(s)} = 0$, then

$$e_\lambda^+(l) = 0.5 e_a \exp(\Gamma_\lambda), \qquad e_\lambda^-(l) = -e_\lambda^+(l)\exp(-2\Gamma_\lambda) \tag{51}$$

Taking only the fastest growing mode, $n = 0$, $\lambda_0 l/V_g \simeq i\pi + \Delta$ (see equation (46)), we have

$$e_\omega^+(l,t) \approx a_0 \frac{\mathcal{E}_{0\omega}\exp(\lambda_0 t)}{1 + R_0 e^{2(\gamma_0 - \lambda_0)l/V_g}} \tag{52}$$

where complex coefficient a_0 is weakly dependent on ω. Let us expand the frequency dependence $T_g(\omega) = 2l/V_g(\omega)$ near the central frequency of a

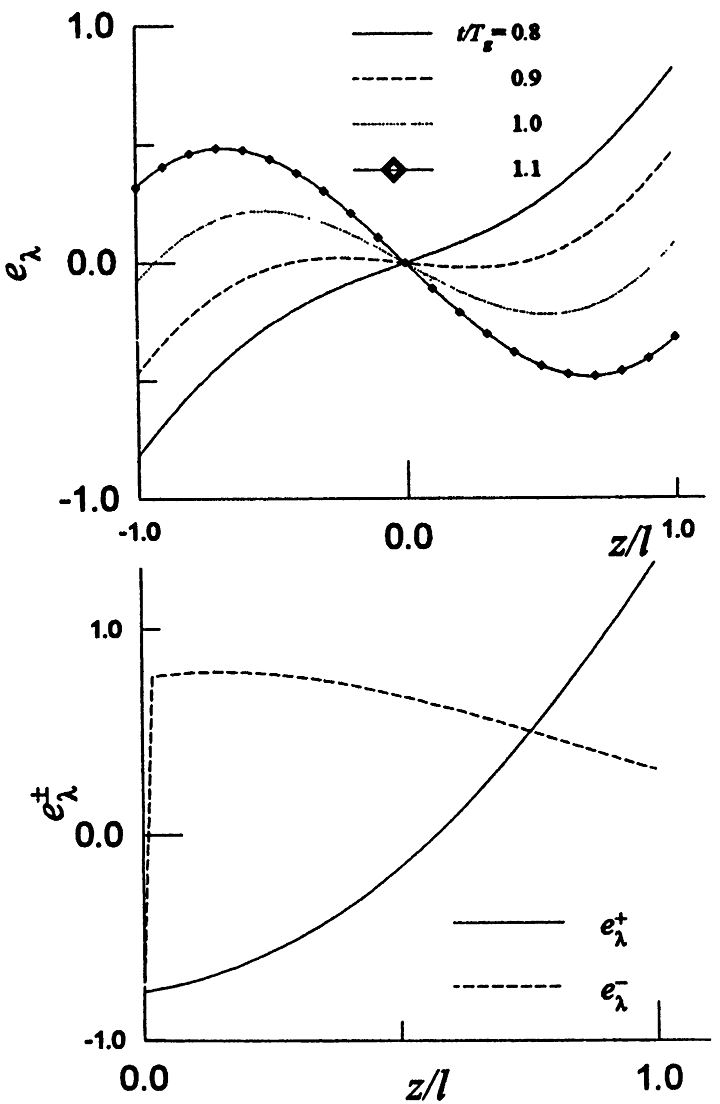

Figure 1: Profiles of the most unstable disturbance (antisymmetric mode, $n = 0$) for different time moments, with exponential growth of the amplitude excluded. *Upper panel:* full disturbance, $e_\lambda = e^+_\lambda e^-_\lambda$; *lower panel:* separate profiles of downward (e^+_λ) and upward (e^-_λ) waves for the right-hand hemisphere; $\mathrm{Re}\Delta_0 = 0.114$, $\mathrm{Jm}\Delta_0 \simeq 0.02$.

packet, $\omega \approx \omega_0$:

$$T_g(\omega) \simeq T_{g0}(\omega) + \left.\frac{\partial T_g}{\partial \omega}\right|_{\omega_0} (\omega - \omega_0) \tag{53}$$

For the sake of simplicity the stationary wave spectrum $\mathcal{E}_{0_\omega}$ (see section 3.1) may be approximated by the Gaussian function

$$\mathcal{E}_{0_\omega} = E_0 \pi^{-1/2} \Delta_\omega^{-1} \exp\left\{-(\omega - \omega_0)^2/\Delta_\omega^2\right\} \tag{54}$$

Then from (52) we obtain

$$\begin{aligned} e^+(l,t) &= |e_0(\omega)| \exp[i\psi(\omega) + (i\pi + \Delta_{01})t/T_g] \\ \psi &= -\mathrm{arctg}\frac{\Delta_{02} + \nu x}{\Delta_{01} + \Delta_a} \\ e_0(\omega) &= \frac{C_0 E_0}{\Delta_\omega \pi^{1/2}} \frac{\exp(-x^2/x_0^2)}{[(\Delta_a + \Delta_{01})^2 + (\nu x + \Delta_{02})^2]^{1/2}} \end{aligned} \tag{55}$$

where $C_0 \propto a_0$, $x = (\omega/\omega_0) - 1$, $\nu = \pi\omega_0 \partial \ln T_g/\partial\omega$, $x_0 = \Delta_\omega/\omega_0$, $\Delta_a = (1 - R_0 e^{2\Gamma_0})_{\omega=\omega_0}$, $\Delta_{0(1,2)}$ are real and imaginary parts of Δ_0, which are determined by (46) at $\nu x_0 < |\Delta|$; it is important that they are independent on ω. Near the center frequency of a wave packet, $|\omega - \omega_0| < \Delta_\omega$, temporal evolution of maximum intensity point is found from the condition of phase stationarity:

$$\psi(\omega) + \pi t/T_g = C \tag{56}$$

In particular, when $\psi \ll \pi$, we obtain

$$\omega_c(t) - \omega_c(0) = \frac{t}{T_{g0}} \frac{\Delta_{01} + \Delta_a}{\partial \ln T_g/\partial\omega} \tag{57}$$

From (46) we know that $(\Delta_{01} + \Delta_a) \propto -\partial R/\partial n_i$. Thus we see that at linear deviations from the stationary solution to the ASM equations the nonlinearity of ionosphere reflection contributes substantially to the evolution of a wave frequency of a Pc 1 packet in presence of a group time dispersion, $\partial T_g/\partial\omega \neq 0$.

4 Discussion

We considered a model for Pc 1 pearls, which attributes their formation to the interaction between magnetosphere and ionosphere systems. Periodic regime of Alfven wave generation and formation of wave packets with frequency drift were related with peculiarities of ion cyclotron instability with account of nonlinearity of the ionosphere reflection coefficient depending on the flux of precipitated energetic protons that give rise to Pc 1 waves. The most straightforward verification of the model would be experimental determination of presence or absence of wave signal reflected from the ionosphere, and investigation of its frequency/time spectrogram in connection with evolution of the ionosphere. Such a study is difficult to perform on the basis of the data reported to date, since satellite recordings of Pc 1 pearls are much more rare than ground-based ones, and they display pearl features not as clearly [*Perraut*, 1982; *Guglielmi et al.*, 1992; *Erlandson et al.*, 1992]. *Erlandson et al.* [1992] analyzed Viking data and found a succession of 7 downgoing pearl packets, the reflected (upgoing) signal being 5–10 times less. This amplitude of reflected waves is small from the point of view of observations, because such signals are difficult to analyze. However, from the point of view of wave generation mechanism, the reflection coefficient of 0.1–0.2 is not small, because the path-integrated gain necessary for the instability is then only $\Gamma \simeq 2.4$–1.3, whereas $\Gamma > 10$ is usually needed for a signal to be amplified from background noise level on one path through interaction region.

In absence of direct verification of the ASM model, based on satellite data, let us address to well documented statistical features of pearls listed in the Introduction. First, we will discuss temporal characteristics of pearls. As we have seen from the theory presented, wave propagation time between conjugate hemispheres, T_g, turns out to be an intrinsic nonlinear timescale for a system including ionospheric mirrors with variable reflection coefficient (see the dispersion relation (43). We should note that wave group period T_g appears also in the nonlinear mechanism connected with quasi-linear modification of the distribution function of energetic particles, discussed by *Bespalov* [1984]. An important consequence of the ASM mechanism is predominance of an antisymmetric shape of wave disturbance relative to the equator, which corresponds to only one oscillating wave packet, with its mirror counterpart suppressed (see section 3.3). This feature, which matches well the observations [*Saito*, 1969], is rather difficult to explain naturally without involving the ionosphere influence into consideration; it disinguishes the ASM model from other mechanisms.

Besides the notion of an oscillating wave packet, also bouncing proton bunches were discussed as possible candidates for the pearl formation [*Jacobs and Watanabe*, 1963]. This mechanism was recently recalled by *Erlandson et al.* [1992], who analyzed a Viking recording of several subsequent pearl packets at L =3.87–4.10 and found, that the period dependence on L-shell was substantially weaker than the calculated dependence of wave transit time, T_g; they considered the proton bounce mechanism as an alternative one, which may fit better the registered L-dependence of the pearl repetition period. We note that pearl wave packets are formed near the plasmapause, and thus they have a finite transverse extent determined by waveguide properties of the plasmapause [*Dmitrienko and Mazur*, 1985, 1992]. According to the latter paper, this extent is of order 10^3 km near the equator, which corresponds to $\Delta L \simeq 0.15$. We see that the observations by *Erlandson et al.* [1992] lie within the limits of transverse extent of an Alfven wave packet; at these distances, generally speaking, the packet transit time does not obey a simple dependence on L, caclulated according to the geometrical optics, rather it should be weaker, so this is one possible explanation of the observations discussed.

Let us make some quantitative estimates of the conditions for the spike-like regime of the ASM. With account of equations (7),(9), and (20), the parameter of nonlinearity of the ionosphere, $\mathcal{R}$, may be written as $\mathcal{R} \sim Q(l/V_g)(\partial R/\partial n_i)\, I_0$, $I_0 = 2S_0 = \pi\sigma \int JT_b\, v^3\, dv\, d\eta$ (J is the source of energetic protons). From the dispersion relation (46) we see that for the instability, $\partial R/\partial n_i$ should be negative, and the growth rate of the disturbance is proportional to $|\mathcal{R}|$. As shown by *Polyakov et al.* [1983]; *Ostapenko and Polyakov* [1990], the negative sign of $\partial R/\partial n_i$ is realized in the morning or evening sectors, where $|\mathrm{Re} Z_i| \gtrsim \mathrm{Jm} Z_i \sim n_A^{-1}$ (see (14)). Typical values of $\partial R/\partial n_i$ are $(2\text{–}3)\cdot 10^{-5}$ cm^3. On the day side $\partial R/\partial n_i$ is rather small. The latter is also true at all local times for periods of high disturbance level when the IAR features are not very pronounced. For proton energy $W_0 = 200$ keV we have $Q \approx 10^{-2}$ cm^{-1}; for proton source flux $I_0 \sim 3 \cdot 10^4$ cm^{-2} c^{-1} we obtain $\mathrm{Re}\lambda \sim 3 \cdot 10^{-3}$ c^{-1}; so for realistic values of parameters, the most unstable disturbance grows during several wave transit periods ($T = 2T_g \sim 100$ s). If we estimate the criterion for instability of adiabatic mode (32), then for the same parameters we get the inequality $\partial R/\partial n_i \gtrsim 10^{-3}$ cm^3; this is rather restrictive condition, so the estimations confirm that in the ASM mechanism a disturbance in the form of a single wave packet oscillating between conjugate ionospheres is the most unstable one.

We have noted above that the properties of the IAR favour the formation of pearl-type Pc 1 emissions at the morningside and the eveningside magnetosphere during quiet periods, when the derivative $\partial R/\partial n_i$ may have sufficiently large negative values. From theoretical point of view, there is no substantial asymmetry between the parameters of the IAR in the morning and the evening sectors; experimental results of the investigation of the IAR characteristics [*Belyaev et al.*, 1989] also did not reveal such asymmetry, although those studies have been performed only at rather low latitudes ($L \simeq 2.65$). There is, however, a well-known asymmetry in the structure of Pc 1 emissions between the morning and the evening sectors [*Guglielmi and Troitskaya*, 1973; *Nishida*, 1978]. To our mind, it is related with the difference in the mechanisms of supply of energetic particles to those sectors at different phases of a storm. Eveningside plasmasphere is subject to injection of energetic protons mainly at the main phase, when the ionosphere is disturbed, and the influence of the nonlinearity of reflection coefficient, discussed in this paper, is not so significant. Accordingly, during main phases of storms Pc 1 emissions, connected with plasmasphere, are recorded predominantly in the evening and are most often of IPDP type [*Pikkarainen et al.*, 1983; *Nishida*, 1978]. One possible mechanism of IPDP formation is quasi-linear dynamics of slow-wave cyclotron maser, discussed by *Bespalov* [1984]. these cyclotron emissions are probably responsible for the evolution of the asymmetric ring current [*Cornwall et al.*, 1970, 1971; *Bespalov et al.*, 1990, 1994]. During periods of low disturbance level the main mechanism, providing the interface of energetic protons with plasmasphere region, is convection and diffusion across the L shells. Under these conditions there are two factors leading to the asymmetry between the morning and the evening sectors. One of them is connected with the direction of combined electric and magnetic drift, which is counter-earthward in the evening and towards the Earth in the morning, thus giving higher acceleration and supply rates of protons at the morning side. The other factor is the asymmetry of plasmapause, which has a bulge in the evening; thus the evening-side projection of the plasmapause is at higher latitudes, where, according to present notions, the oscillating frequency dependence of ionosphere reflection coefficient due to the influence of the IAR is less pronounced, becasue the IAR is disturbed by high precipitation fluxes at subauroral and auroral latitudes. However, the detailed knowledge of the dependence of the IAR properties on latitude and longitude is still absent, so further experimental investigations will provide a test for the considerations presented above.

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Appendix

The formal solution to the linearized and Laplace-transformed equation for wave energy (5) is written as

$$e_\lambda^\pm(z,\omega) = \left[e_{\lambda\,L}^\pm \pm \int_0^z \gamma_\lambda \mathcal{E}_0^\pm\, e^{\mp G_\lambda(z')}\, dz'\right] e^{\pm G_\lambda(z)} \tag{58}$$

where $G_\lambda(z) \equiv G_\lambda(0,z) = \int_0^z (\gamma_0 - \lambda)\, dz/V_g$, $e_{\lambda\,L}^\pm = e_\lambda^\pm(0,\omega$.

The linearized boundary conditions for waves (12) have the form

$$e_\lambda^\pm(\mp l) = R_0^\mp e_\lambda^\mp(\mp l) + \frac{\partial R^\mp}{\partial n_i} n_\lambda^\pm \mathcal{E}_0^\mp(\mp l) \tag{59}$$

and the linearized ionosphere ionization equations are

$$n_\lambda^\pm(\lambda + \nu_r^\pm) = Q S_\lambda^\pm \tag{60}$$

where $\nu_r^\pm = 2\chi n_0^\pm$, S_λ is the Laplace component of the precipitated flux variation. Equations (59) and (60) together give:

$$e_\lambda^\pm(\mp l) = R_0^\mp e_\lambda^\mp(\mp l) + Q\frac{\partial R^\mp}{\partial n_i}\frac{S_\lambda^\mp}{\lambda + \nu_r^\mp}\mathcal{E}_0^\mp(\mp l) \tag{61}$$

We omit a lengthy procedure of calculating S_λ via e_λ, N_λ. The final result is:

$$S_\lambda^\pm = \frac{1}{\lambda + T_0^{-1}}\left[\lambda \hat{e}_\lambda^\mp \pm \Lambda_0^- \hat{e}_\lambda^- \mp \Lambda_0^+ \hat{e}_\lambda^+\right] \tag{62}$$

where $\hat{e}_\lambda^\pm = \int \phi_{10} e_\lambda^\pm\, dz\, d\omega$, and $\Lambda_0^\pm = S_0^\pm/N_0$ are the stationary precipitation rates. For symmetric ionospheres we have $\Lambda_0^+ = \Lambda_0^- = T_0^{-1}/2$, T_0 is the particle lifetime at stationary state.

Let us further concentrate on the case of symmetric ionospheres. Substituting (62) and (58) into (61) and performing the necessary transformations, one gets two equations for wave amplitude variations:

$$e_{\lambda L}^{(a)} = Q\frac{\partial R}{\partial n_i}\frac{\mathcal{E}_{0L} + A_0}{\lambda + \nu_r}\frac{e^{\Gamma_0 + \Gamma_\lambda}}{1 + Re^{2\Gamma_\lambda}}\int \phi_{10} e_\lambda^{(a)}\, dz\, d\omega \tag{63}$$

$$\begin{aligned} e_{\lambda L}^{(s)} = {} & Q\frac{\partial R}{\partial n_i}\frac{\mathcal{E}_{0L} + A_0}{\lambda + \nu_r}\frac{e^{\Gamma_0 + \Gamma_\lambda}}{1 - Re^{2\Gamma_\lambda}}\frac{\lambda}{\lambda + T_0^{-1}}\int \phi_{10} e_\lambda^{(s)}\, dz\, d\omega \\ & + \frac{N_\lambda}{N_c}\left[(g_+\mathcal{E}_{0L} + A_+)R_0 e^{\Gamma_0 + \Gamma_\lambda} + g_-\mathcal{E}_{0L} - A_-\right] \end{aligned} \tag{64}$$

where

$$g_\pm = \Omega_{BL}\int_0^l \phi_2(z) e^{\pm\Lambda(z)}\, dz/V_g$$
$$A_\pm = \Omega_{BL}\int_0^l \varphi_2(z) e^{\pm\Lambda(z)} \int_0^z a_\omega e^{\mp G_0(z')}\frac{dz'}{V_g}\frac{dz}{V_g}$$

$\Lambda(z) = \int_0^z \lambda\, dz/V_g$, $A_0 = \int_0^l a_\omega e^{-G_0(z')} \frac{dz}{V_g}$. The equation for N_λ may be obtained by linearizing (19):

$$N_\lambda = \frac{\int \phi_{10} e_\lambda^{(s)}\, dz\, d\omega}{\lambda + T_0^{-1}} \tag{65}$$

We see that symmetric and asymmetric disturbance modes are independent, and that the asymmetric disturbance does not depend on variations of the energetic particle content.

For further calculations let us perform the integration over z in (63) and (64) With account of (58) we get:

$$\begin{array}{l} \int_{-l}^{l} \phi_{10} e_\lambda^{(a)}\, dz = g_{\omega\lambda} e_{\lambda L}^{(a)} \\ \int_{-l}^{l} \phi_{10} e_\lambda^{(s)}\, dz = g_{\omega\lambda} e_{\lambda L}^{(s)} + \frac{N_\lambda}{N_c} h_{\omega\lambda} \end{array} \tag{66}$$

where

$$\begin{array}{rcl} g_{\omega\lambda} & = & \int_{-l}^{l} \phi_{10} e^{-G_\lambda(z)} dz \\ h_{\omega\lambda} & = & \int_{-l}^{l} \phi_{10} \int_0^z \mathcal{E}_0(z') e^{-G_\lambda(z')} \frac{dz'}{V_g}\, e^{G_\lambda(z)} \frac{dz}{V_g} \end{array}$$

Finally, putting equations (63), (64), (65), and (66) together, we get the linear equations (35).

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